

REPORT

France-Japan Cross-Cultural Research on Role of the Figures in Geometry -Differences and Similarities of Students' Productions between Each Country-

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The purpose of this research is to clarify differences and similarities of students' productions in the proof-problem solving between France and Japan. The following conclusions were obtained from the experimentations: (1) The differences between French and Japanese students' productions were revealed as "directions of base of triangles", "senses of triangles", "types of inferences", and so on. (2) The similarities between French and Japanese students' productions were revealed as "shape of triangles", "actions for figures", and so on. (3) In France, there were two kinds of students' productions which depended or did not depend on their mathematics textbooks. On the other hand, in Japan, almost all of students' productions depended on their mathematics textbooks. It seemed that the fact was influenced by mathematics teachers' beliefs for their teaching and students beliefs for their mathematics learning depending on their mental cultures in each country.

KEY WORDS: Proof, Problem-Solving, Geometry, Cross-Cultural Research

Introduction

This paper is a research report on France-Japan Collaborative Research in the geometrical problem solving. We have focused on students' activities in the proof-problem solving in Geometry. We have clarified characteristics of students' difficulties and have developed the means of helping students who had such difficulties in their problem solving (Harada, Nohda & Gallou-Dumiel, 1993;1997).

We have carried out "cross-cultural research" on the role of the figures in the geometry since 1997. Because we have seen some cross-cultural characteristics in their productions in our previous researches, for example, students' beliefs, attitudes, and practices for their problem solving depending on the cultural characteristics in each country.

We had already made the experimentations with a problem which could be solved by a common procedure, that is, "conditions of congruent triangles", for students in France and Japan, and had clarified the differences and similarities of students' productions in the proof-problem solving between each country (Gallou-Dumiel, Harada & Nohda, 1997).

The purpose of this research is to clarify differences and similarities of students' productions in the proof-problem solving between France and Japan based on experimentations with a problem. The problem can be solved by different procedures between students in each country, "rotations" in France and "conditions of congruent triangles" in Japan.

Methods of Research

1) Problem

Problem: ABC is a triangle. Triangle P_1AB and P_2AC are equilateral.

(P_1 and P_2 are points of outside of triangle ABC.)

Prove $P_1C=P_2B$.

This problem is given in common with Secondary School Curriculum in each country, that is, the problem is given in Lycée 1^{ères} in France and in Junior High School, the 2nd Grade in Japan. It is a typical problem in geometrical proof-problems in each country. The problem is given to students as a type of written problem without geometrical figures because we have the aim of considering students' knowledge of not only proofs but also constructions of figures in geometry¹⁾.

2) Subjects

In France: 20 students in Lycée Champollion (Grenoble) 1^{ères}. These students had Terracher Mathematics Textbook.

In Japan: 20 students in Junior High School (Tsukuba-shi) 3rd Grade. These students had Dainihon-Tosho Mathematics Textbook.

3) Methods

We directed students firstly to construct a geometrical figure which is expressed in the problem and secondly to prove a proposition in it. They could spend about 40 minutes to solve the problem.

In consideration of results of experimentations, we selected 20 students' productions which could be produced by using "rotations" in France and 20 students' productions which could be produced by using "conditions of congruent triangles" in Japan.

We interviewed the teacher about the way he teaches geometry in each country.

4) Viewpoints of Experimentations

We chose the variables and values in the experimentations in anticipations of students' productions in both of countries.

Variables for Drawings of Triangle

We decided to study a problem of the "prototype examples" for triangles²⁾. From this point of view, we chose the two variables for drawing of triangles, that is, "shapes of triangle" and "directions of base on triangle".

| variables | values |
|---------------------|---|
| shapes of triangles | { acute-angled triangles obtuse-angled triangles right-angled triangles |
| directions of base | { horizontal non horizontal |

Variables for Actions for Figures

We chose the variables for "actions for figures". In geometrical proof-problem solving, to conjecture process of proof, it is necessary for students to discover "procedures for proof". It will be discovered through "actions for figures". For example, the "actions for figure" mean the actions which they produce when they part a complex figure into elementary ones. Then we especially focused on the three variables, that is, "marking segments", "marking angles", and "marking figures", taking into account of this problem solving.

| variables | values |
|------------------|-------------|
| marking segments | { yes no |
| marking angles | { yes no |
| marking figures | { yes no |

Variables for Senses of triangles

We decided to consider students' recognitions of the "senses of triangles"³⁾. Then we focused on "direct sense"(positive direction) and "indirect sense"(negative direction) as shown "Figure 1".

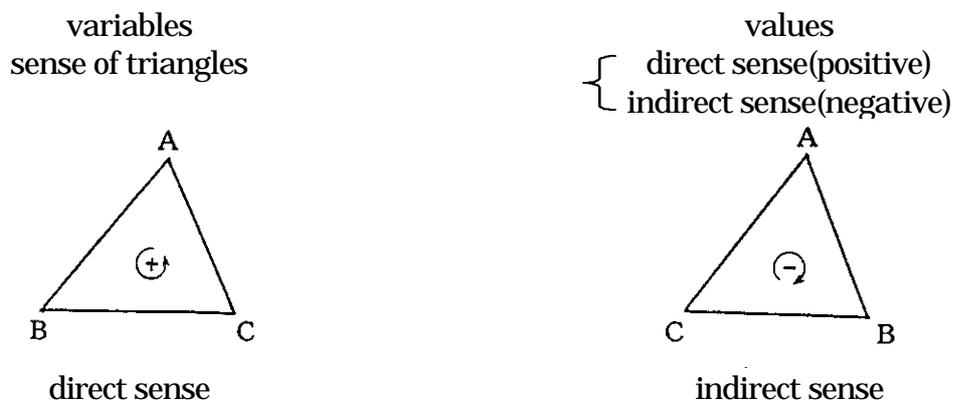


Figure 1. Illustrations of Senses of Triangles

Variables for Types of Inferences

We chose the variables for types of inferences. To consider to types of inferences which students used in their proof processes, we focused on what kinds of "conjunctions" were used in the proof processes.

We especially focused on two types of inferences relating to "conjunctions" as follows:

Type 1 (saying reason in advance): This inference is used when, in case of saying a new proposition, one says reasons for the propositions in advance. We especially focused on "Par....., ~", "Pusique....., ~", and "Comme....., ~" in France and "だから" ("Since....., ~") and "より" ("By, ~").

~") in Japan in proof processes.

Type 2 (saying reasons in later): This inference is used when, in case of saying a new proposition, one says reasons for the proposition in later. We focused on "..., par ~", "....., puisque ~", and ".....()" in France and "なぜなら"("....., because ~ ") and ".....()" in Japan in proof processes.

| | |
|---------------------|--|
| variables | values |
| types of inferences | { Type 1 (saying reasons in advance) { Type 2 (saying reasons in later) |

Variables for Type of Proof Writing

We chose the variables for the types of proof writing. We focused on the two types of proof writing, that is, the types of expressions based on "the model of proof writing" in their mathematics textbooks and based on their "thinking processes" in the problem solving.

| | |
|------------------------|--|
| variables | values |
| types of proof writing | { based on the model of proof writing { based on thinking processes |

Variables for the Kinds of Errors in Proofs

We chose the variables for the four kinds of students' errors in proofs.

| | |
|---------------------------|---|
| variables | values |
| kinds of errors in proofs | { faults of conditions { addition of conditions { errors of inferences { careless mistakes |

Results of Experimentations and Considerations

Drawings of Triangle

| | | |
|-----------------------------------|--------|-------|
| • Shapes of Triangle | France | Japan |
| acute-angled triangle | 16 | 16 |
| (equilateral triangle | 0 | 1) |
| (isosceles triangle | 0 | 1) |
| obtuse-angled triangle | 4 | 2 |
| right-angled triangle | 0 | 2 |
| • Directions of Base on Triangles | France | Japan |
| horizontal | 14 | 17 |
| non horizontal | 6 | 3 |

Actions for Figures

| | | |
|---------------------|--------|-------|
| Actions for figures | France | Japan |
| marking segments | 14 | 18 |
| marking angles | 13 | 16 |
| marking figures | 0 | 2 |

Senses of Triangle

| Senses of triangle | France | Japan |
|--------------------|--------|-------|
| direct sense | 14 | 19 |
| indirect sense | 6 | 1 |

Types of Inferences

| Type 1 (saying reasons in advance) | France | Japan |
|------------------------------------|--------|-------------------|
| "Par..., ~" | 8 | "By..., ~" 16 |
| "Pusique ..., ~" | 0 | "Since ..., ~" 19 |
| "Comme ..., ~" | 4 | |
| Type 2 (saying reasons in later) | France | Japan |
| "..., par ~" | 6 | "...,because ~" 0 |
| "..., car ~" | 5 | ".....()" 7 |
| "...,pusique ~" | 1 | |
| ".....()" | 0 | |

Type of Proof Writing

| Type of Proof Writing | France | Japan |
|----------------------------------|--------|-------|
| based on models of proof writing | 12 | 16 |
| based on thinking processes | 6 | 0 |
| (otherwise | 2 | 4) |

Kinds of Errors in Proofs

| Kinds of Errors in Proofs | France | Japan |
|---------------------------|--------|-------|
| fault of conditions | 2 | 2 |
| addition of conditions | 0 | 1 |
| errors of inferences | 0 | 1 |
| careless mistakes | 2 | 0 |

Considerations of Results of Experimentations

Considerations of Results

We considered the results of experimentations based on the comparisons of the Mathematics Textbooks between France and Japan(See "Appendix 2") from the points of view of cross-cultural research.

1) Drawings of Triangle

On "shapes of triangles", French students drew acute-angled triangles or obtuse-angled triangles. Then we think that they could recognize the generality of triangle in this problem. On the other hand, some Japanese students drew the special triangles(equilateral triangle and isosceles triangle). We think that they could not recognize the generality of triangle in this problem.

Many students in each country drew "acute-angled triangles". The shapes of triangles depended on their mathematics textbooks in each country. We could also consider that the students in each country drew acute-angled triangles in order to represent generality of triangles in this problem. Then we think that they realized the acute-angled triangles as a representation about generality of triangles.

On "directions of base on triangles", in French students, there were two kinds of directions of base on triangles: horizontal or non horizontal. On the other hand, almost all of Japanese students drew horizontal bases of triangles only. We think that the students' productions in each country depended on the characteristics of their mathematics textbooks.

We could also focus on the fact as follows: Almost all of Japanese students drew acute-angled triangles which had horizontal bases. On the other hand, French students drew acute-angled or obtuse-angled triangles which had non horizontal bases. We then think that almost all of Japanese students had "prototype example" of triangles, whereas almost all of French students had not "prototype examples" of triangles.

2) Actions for Figures

There was few difference in the number of students who had "marking segments" as well as "marking angles" between France and Japan. However, there was the difference in the number of students who had "marking figures" between each country.

Although there were few "marking segments" and "marking angles" in their mathematics textbooks in France, almost all of the students had such two kinds of "actions for figures". We think that the French students' actions in the problem solving depended on their lessons on everyday. On the other hand, Japanese students' actions for figures strongly depended on the geometrical figures in their mathematics textbooks.

3) Sense of Triangle

In France, there were one kind of sense ("direct sense") in their mathematics textbooks. However, there were two kinds of senses of triangles ("direct sense" or "indirect sense") in their productions in the experimentation. Then French students were not influenced by their textbooks.

On the other hand, in Japan, almost all of the students produced only one type of sense ("direct sense") of triangles. We think that they strongly depended on the figures of triangles in their mathematics textbooks.

4) Types of Inferences

For "Type 1" of inference(saying reasons in advance), few French students used the terms relating to the inference, whereas almost all of Japanese students used the terms. The students who had "Type 1" of inference depended on the characteristics of their mathematics

textbooks in each country.

In "Type 2" of inference(saying reasons in later), a few French students used the terms relating to the inference, whereas few Japanese students used the terms but ".....()". The results of the experimentation in France were influenced by the characteristics of their mathematics textbooks. On the other hand, the results of the experimentation in Japan slightly depended on their mathematics textbooks.

5) Type of Proof Writing

In France, there were the two types of proof writing, that is, based on the model of proof writing in their mathematics textbooks and based on their thinking processes in the problem solving. The results of experimentation in France did not depend on the proof writing of their mathematics textbooks.

On the other hand, almost all of Japanese students used only one type of proof writing, that is, based on the model of proof writing. The results of experimentation in Japan were influenced by their mathematics textbooks.

6) Kinds of errors in proofs

In the kinds of errors in proofs, the "faults of conditions" were produced by students in each country. The "addition of conditions" and "errors of inferences" were produced by students in Japan only. The "careless mistakes" were produced by students in France only.

We think that French students were making progress with their logical thinking in comparison with Japanese students. It seemed that the fact depended on the differences in their cognitive developmental levels, attitude for mathematics learning on every day, or School Mathematics Curriculum between France and Japan.

7) Relationships between drawings of figures and errors in proofs

We considered the relationships between kinds of drawings of figures and kinds of errors in proofs which were produced by students. The Japanese students who had errors in proofs had drawn the special triangles(equilateral triangles,isosceles triangles and right angles triangles). On the other hand, the French students who had errors in proofs had drawn general triangles(acute-angled triangles).

We think that there were a relationship between the shapes of drawings triangles and the kinds of errors in proofs in Japanese students. On the other hand, there were less relationships between the shapes of drawings of figures and the kinds of errors in proofs in French students. It seems that the several Japanese students could not reach the logical thinking level which they could recognize the inferences of proof in the problem, whereas French students could reach such level but forgot to describe the conditions in the proofs carelessly.

We also think that the results are related to the fact that Japanese students had "prototype example" of triangles, whereas the majority of French students does not have one and this helps the French students firstly in reaching the generality of figures and secondly in seeing elements of figures.

Some Findings in Considerations of Results

We will insist that there appeared some findings in the considerations of the results of experimentations.

- 1)** In France, there were the two kinds of students' productions which depended or did not depend on their mathematics textbooks . On the hand, many Japanese students' productions depended on their mathematics textbooks.
- 2)** In France, the students' productions which depended on their mathematics textbooks were "drawing of figures"(shapes of triangles; directions of base of triangle), and "type of inference"(saying reasons in advance; saying reasons in later).
- 3)** In Japan, the students' productions which depended on their mathematics textbooks were "drawing of figures"(shapes of triangles; directions of base of triangle), "actions for figures"(making segments; marking angles;marking figures), "sense of triangle", "types of inference"(saying reasons in advance), and "type of proof writing".
- 4)** In France, the students' productions which did not depend on mathematics textbooks were "actions for figures"(marking segments; marking angles) and "types of proof writing".
- 5)** In Japan, the students' productions which did not depend on their mathematics Textbooks were "saying reasons in advance".
- 6)** The differences between French and Japanese students' productions were revealed as "directions of base of triangles", "sense of triangles", "types of inferences (saying in advance; saying in later),"type of proof writing", and "types of errors in proofs"(addition of conditions; errors of inferences; careless mistakes).
- 7)** The similarities between French and Japanese students' productions were revealed as "shape of triangles", "drawings acute-triangles as presentation of generality of triangles", "actions for figures"(marking segments; marking angles), and "type of errors of proofs"(faults of conditions).
- 8)** There was a relationship between the shapes of triangles and the kinds of errors in proofs which were produced by Japanese students. On the other hand, there were less relationships between the shapes of figures and the kinds of errors in proofs which were produced by French students. We think that the results were related to the fact that Japanese students had "prototype example" of triangles, whereas almost all French students did not have one.

9) From viewpoints of cross-cultural research, firstly it seemed that "prototype examples" of triangles depended on Mathematics Curriculum or the teachers' teaching methods on everyday in Japan. We also think that the "prototype examples" were influenced by Japanese mental culture relating to drawings of geometrical figures from the viewpoints of seeking stability of figures.

Secondly we could consider again the fact that there were two kinds of students' productions which depended or did not depend on their mathematics textbooks in France, whereas many Japanese students' productions depended on their mathematics textbooks. We think that the fact was influenced by the mathematics teachers' beliefs for their teaching and students' beliefs for their mathematics learning depending on their mental culture in each country. Especially, in France, the teachers generally are independent of the mathematics textbooks. Because they think there is not enough things in the textbooks and teach their students various ideas and ways of problem solving. The students work on what their teacher says and not on what is written in their textbooks. Then it is easier for students to be independent of their textbooks and to think by their own.

Conclusions

From the considerations of results of experimentations, the following conclusions were obtained:

- 1)** In France, there were two kinds of students' productions which depended or did not depend on their mathematics textbooks. On the other hand, many Japanese students' productions depended on their mathematics textbooks.
- 2)** The differences between French and Japanese students' productions were revealed as "direction of base of triangles", "sense of triangles", "types of inferences"(saying in advance; saying in later), "type of proof writing", and "types of errors in proofs"(addition of conditions; errors of inferences; careless mistakes).
- 3)** The similarities between French and Japanese students' productions were revealed as "shape of triangles"(acute-angled triangles), "drawing of acute-angled triangles as a presentation of generality of triangles", "actions for figures"(marking segments; marking angles), and "type of errors of proofs" (faults of conditions).
- 4)** There were a relationship between the shapes of triangle and the kinds of errors in proofs which were produced by Japanese students. On the other hand, there were less relationships between the shapes of triangles and the kinds of errors in proofs which were produced by French students.
- 5)** From viewpoints of cross-cultural research, we could focus on the two facts as follows: Firstly,

the Japanese students had the "prototype examples" of triangles. It seems that their "prototype examples" were influenced by Japanese mental culture. Secondly, there were two kinds of French students' productions which depended or did not depend on their mathematics textbooks, whereas many Japanese students' productions depended on their mathematics textbooks. We think that the fact was influenced by their mathematics teachers' beliefs for teaching and students' beliefs for their mathematics learning depending on their mental culture in each country.

It is necessary for us to consider our conclusions from points of views on France and Japan cross-culture in detail, through the interviews to teachers and their students as a case study and the experimentations for a lot of students as a statistical study in the future.

Notes

- 1) The examples of geometrical figures which were drawn by students in the experimentations are shown in "Appendix 1".
- 2) We identified "prototype example" of triangles as the acute-angled triangle which had a horizontal base, in the students' concept images. (Gallou-Dumiel, et al, 1997)
- 3) There are two kinds of transformations which preserve or do not preserve "the sense of triangles". The characteristics is very important in Transformations.

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Appendix 1

Examples of geometrical figures which were drawn by students

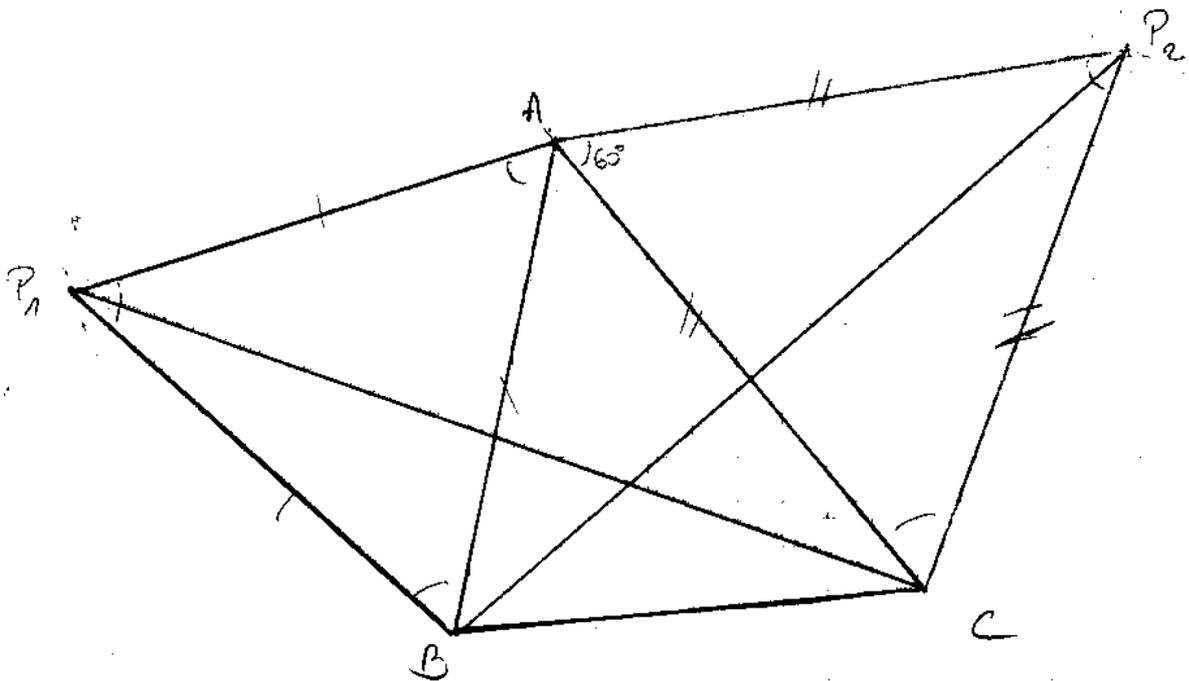


Figure which was drawn by French student

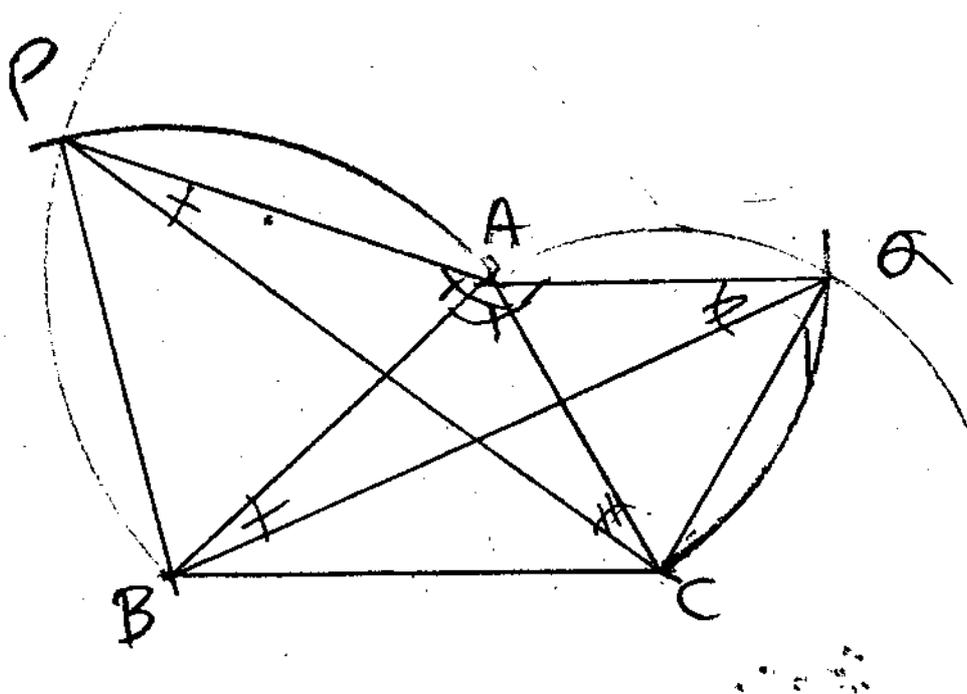


Figure which was drawn by Japanese student

Appendix 2

Comparison of Mathematics Textbooks between France and Japan

We chose 40 problems which were described in Mathematics Textbooks in each country. That is, in France, 20 problems in each of 3^e Textbook and 2^e Textbook were chosen. In Japan, 20 problems in each of Junior High School, 2nd and 3rd grade Textbooks were chosen. We made comparisons 40 problems between each country from the points of view of our experimentations. The lists of these textbooks are shown in the references.

The numerals in the table show the number of problems which correspond to each item but the number of triangles show the number of all triangles in the problems.

| | | | | |
|-----------------------------------|--------|--------------------|----|--|
| • Shapes of triangle | France | Japan | | |
| acute-angled triangle | 17 | 27 | | |
| right-angled triangle | 8 | 10 | | |
| obtuse-angled triangle | 4 | 3 | | |
| • Directions of base on triangles | France | Japan | | |
| horizontal | 18 | 28 | | |
| non horizontal | 11 | 12 | | |
| • Actions for Figures | France | Japan | | |
| marking segments | 1 | 9 | | |
| marking angles | 1 | 18 | | |
| marking figures | 3 | 3 | | |
| • Senses of triangles | France | Japan | | |
| direct sense | 17 | 14 | | |
| indirect sense | 1 | 1 | | |
| • Types of Inferences | | | | |
| saying reasons in advance | France | Japan | | |
| "Par..., ~" | 5 | "By..., ~" | 30 | |
| "Pusique ..., ~" | 2 | "Since..., ~" | 23 | |
| "Comme..., ~" | 11 | | | |
| saying reasons in later | France | Japan | | |
| "..., par ~" | 9 | "...,because ~" | 0 | |
| "..., car ~" | 0 | ".....()" | 0 | |
| "..., pusique ~" | 2 | | | |
| ".....()" | 8 | | | |