Article

# Mathematical Proof, Argumentation, and Classroom Communication:
# From a Cultural Perspective

Yasuhiro SEKIGUCHI

Yamaguchi University

*This paper is a contribution to a recent discussion about relationship between mathematical proof and argumentation. It argues from a cultural perspective. First, communication styles in Japanese culture are described, compared with those of Western culture. Then, argumentation and mathematical proof in Japanese schools are examined, focused on how they are related to the general styles of communication in Japanese culture. Finally, it is argued that it would be **culturally** difficult for Japanese to actualize the argumentation in the classroom, and also to teach mathematical proof as argumentative activity.*
KEY WORDS:    proof, argumentation, communication, culture

Since Imre Lakatos presented a dialectical view of the development of mathematical knowledge in his *Proofs and Refutations* (1976), the idea of actualizing dialectic processes in mathematics classroom has often attracted attention of mathematics educators (for a critical review, see Hanna, 1996). Some researchers investigated possibilities of locating mathematical proof in the context of classroom argumentation, that is, in the exchanges of conjectures, explanations, justifications, and refutations among students (Balacheff, 1987, 1991a, 1991b; Sekiguchi, 1991).

Recently, Balacheff suggested difficulties of employing argumentation in the context of mathematical proof, and pointed out that there is an epistemological gap between argumentation and mathematical proof: "argumentation constitutes an epistemological obstacle to the learning of mathematical proof, and more generally of proof in mathematics" (Balacheff, 1999).

Researchers (Boero, 1999; Douek, 2000; Duval, 1999) have continued to discuss complex relationship between mathematical proof or proving and argumentative activity. The present paper is going to join this discussion from a cultural perspective, using Japanese culture as a case. I am going to show that considerations on cultures of the society and classroom are necessary to actualize the learning of mathematical proof in argumentative activities.

For this purpose I propose the concept of communication as an overall framework (Figure 1). This is because I consider that (1) cultural aspects are well-reflected in the style of communication, (2) argumentation is a kind of verbal communication, and (3) mathematical proof is an important component of communication in the mathematical community.
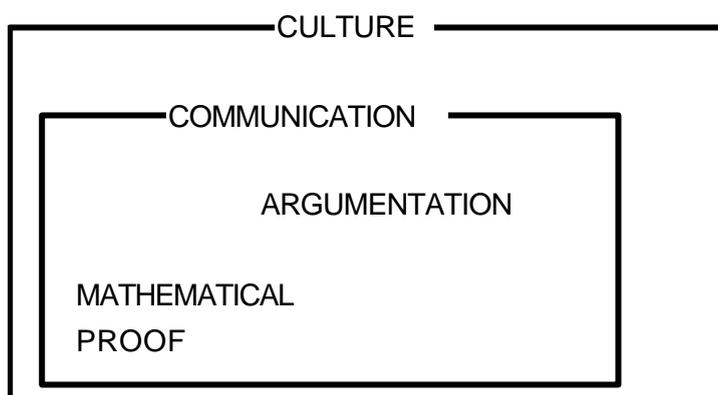
Yasuhiro SEKIGUCHI

```
┌─────────────────CULTURE ─────────────────┐
│                                           │
│   ┌─────────────COMMUNICATION ──────────┐ │
│   │                                      │ │
│   │          ARGUMENTATION               │ │
│   │                                      │ │
│   │   MATHEMATICAL                       │ │
│   │   PROOF                              │ │
│   └──────────────────────────────────────┘ │
└─────────────────────────────────────────────┘
```

*Figure 1.* An overall framework.

The term "culture" I use here is the one of anthropology. Especially, I am going to rely on a cognitive anthropology's perspective. I consider a culture to be "a very large and heterogeneous collection of *models* or... *schemas*" (Shore, 1996, p. 45; also see D'Andrade, 1995). This perspective is very useful for analyzing cognitive aspects in cultural setting.

In the following, I first describe communication styles in Japanese culture, comparing with those of Western culture. Then, I discuss argumentation and mathematical proof in Japanese schools, focusing on how they are related to general styles of communication in Japanese culture.

## Communication and Argumentation in Japanese Culture

Barnlund (1975) pointed out that Japanese traditional culture does not always place the highest value on verbal communication in the communicative activity (pp.129-138, see Table 1). The goal of communication in public is a harmony ("wa") among the participants. Difference between opinions among the participants is conceived of as a threat to the harmony. Therefore, people tend to avoid explicit expression of disagreement in public. The harmony is often symbolized by uniformity or homogeneity in appearance, behaviors, expressions, and so on, within a community. The community emphasizes social obligations ("gimu," "giri," "tatemae") of the community. Cooperation rather than competition is highly valued within a community. Therefore, a person who deprive from the community's obligations sometimes receive rather emotional reactions–e.g., accusation, isolation, or expulsion--than rational ones. It is well-known that even in academic conferences, Japanese do not openly argue with each other very much. Expressing direct opposition is considered impolite: Opposition is usually indirectly or euphemistically expressed (Nakayama, 1989). This communication style of Japanese may be called the "group" model (Befu, 1980).

Table 1

*Different Communication Styles between Japanese and Americans*

|  | Japanese | Americans |
| --- | --- | --- |
| GOAL | social harmony ("wa") | valid conclusions |
| SUBJECT MATTER | interpersonal attitudes (human relations) | individual ideas |
| MEANS | intuition based on shared backgrounds | verbal argument |
| DIFFERENCES OF OPINION | to be avoided | highly valued |

Individuals do not always agree with each other, of course, in any culture. They need to have opportunities to express their own opinions and negotiate them. The group model does not describe those opportunities. As Befu points out, there is a complementary model of Japanese communication, where individuals exchange their spontaneous opinions and feelings (Befu, 1980). It is called a "social exchange" model ("tsukiai"). In this model, people try to provide their own resources (e.g., information, knowledge, skills, power, wealth, friends) to others who need them and are expected to give generous returns (Moeran, 1984, p. 254). In informal opportunities like private talk with close friends or conversation in a drinking party of coworkers, people express rather openly their natural opinions and feelings ("ninjo," "honne"), and negotiate them.

This model is able to capture the communication within a personal network across groups or communities. It emphasizes expressing and sharing of individual's spontaneous opinions, feelings, interests, and the like, rather than the loyalty to the group. Therefore, there is an inherent tension between the two models, which causes conflicts between social obligations and personal relationship.

## Argumentation in Western Culture

For the process of exchanging opinions in Western culture, Toulmin (1958) described a pattern of argument ("Toulmin model", see Figure 2), consisting of four components: claim, grounds, warrant, and backing. A claim is a single statement advanced for adherence of others. Grounds are the specific facts relied on to support a given claim. Warrant is a general statement that justifies the use of the grounds as a basis for the claim. Backing is any support that provides more specific data for warrant:
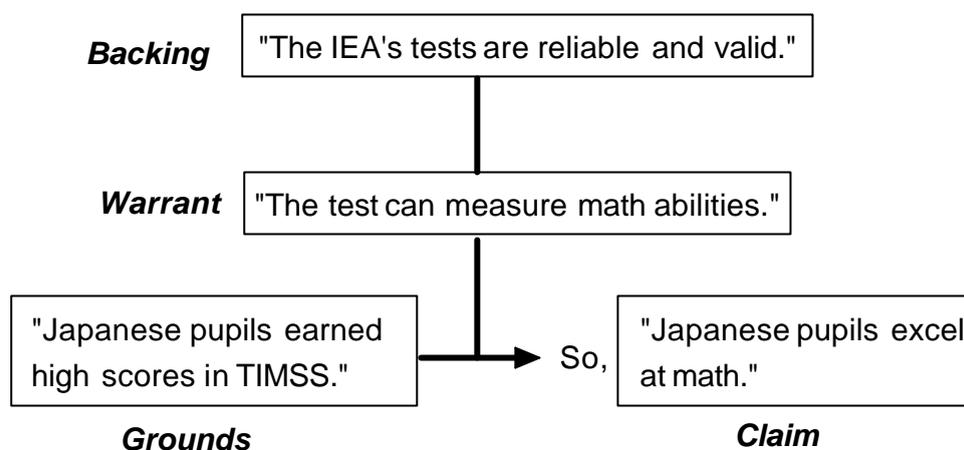
*Figure 2.* Toulmin model of argument.

The argument is embedded in the human interaction of argumentation: "the whole activity of making claims, challenging them, backing them up by producing reasons, criticizing those reasons, rebutting those criticisms, and so on" (Toulmin, Rieke & Janik, 1984, p.14). These components of the model are "armaments" of Western style argumentation. As Lakoff and Johnson (1980) suggest, a "war" metaphor underlies this argumentation:

Many of the things we *do* in arguing are partially structured by the concept of war. Though there is no physical battle, there is a verbal battle, and the structure of argument--attack, defense, counterattack, etc.--reflects this. (Lakoff & Johnson, 1980, p. 4)

In contrast, in Japan, exchanging talks in either public or private is usually referred to as "hanashi-ai": The word means conversation or mutual consultation, and does not signify a war. The top priority in "hanashi-ai" is not logic but human relations. Because people try to avoid direct confrontation, they try to put their opinions *ambiguously* so that they can withdraw or change them easily when others indicate oppositions. As a result, people in "hanashi-ai" do not usually bring up such full logical defense devices like "grounds," "warrants," and "backing." Even in those situations where the social exchange model is working, people tend to avoid bringing up logical armaments because they feel that arguing logically is impersonal ("katakurushii"). In ordinary life, logic ("ronri") is often equated with "rikutsu." The latter is often used derogatorily. Argument which emphasizes "rikutsu" is considered superficial and not reaching the audience's hearts. Analyzing Japanese cultural practices, Moeran (1984) pointed out that Japanese put more value on "kokoro" (hearts) than rational thinking:

Western culture and Western technology are seen by the Japanese as products of Cartesian rationalism. They have revered Western culture and technology over the past century, only to discover that both leave much to be desired. There is something 'missing'; the Western mind is too limited in its horizons and the Japanese mind feels that it has grasped all that can be offered by the West and has room to spare. This is where *kokoro*

comes in. It is essentially a Japanese concept which, it is claimed, understands beyond mere 'intellectualism'. (p. 263)

Thus, even in the social exchange model, logical argumentation is not preferred (see also  Kitade, 1993).

## Argumentation and Proof in Japanese Classrooms

In Japanese schools, the classroom processes contain both formal and informal opportunities, in the sense of the general Japanese culture. Classroom lessons usually contain processes of exchanging opinions in a whole-class or small-groups. These processes are again called "hanashi-ai" as in the adult societies. Roughly speaking, "hanashi-ai" in a whole-class seems to follow the group model, and "hanashi-ai" in small-groups seems to follow the social exchange model.

### *"Hanashi-ai"*

As mentioned already, confronting someone's argument in public is not encouraged in Japanese culture: Opposition is usually indirectly or euphemistically expressed. In school, children are not totally socialized to the adult culture from the outset, however. They may directly express opposition or disagreement in classroom talks, and may endanger the classroom harmony. The classroom teacher plays an important role here. In general, the teacher expresses respects to individual children's ideas, whether they are wrong or not. The teacher tries to use a conflict between children's claims as a good opportunity to deepen children's understanding of the issue in question. That is, the teacher handles the conflict not just as a problem between the involved children but instead frames a problem of the *whole* class from it: The conflict is shared among the classroom participants, and becomes "our" problem (cf. Lewis, 1995, pp. 125-130). The teacher encourages the whole class to think about it and give suggestions. All the class members are supposed to work together towards resolution of the problem, so that the reached resolution produces a recovery of the harmony in the classroom community.

Japanese teachers pose sometimes open-ended problems in the opening of lesson (e.g., Becker et al., 1990). They encourage children to present their own ideas for solving the problems. In a lesson, a teacher asks children to do "hanashi-ai" in small groups, or a whole class. Children often make wrong conjectures and ideas, and procedural mistakes. Also, children may produce several different solutions. The teacher encourages them to compare their ideas and solutions with each other. At those occasions, counterexamples may be found, and counterarguments may occur. The teacher intentionally use such opportunities to stimulate children's thinking. Japanese traditional discipline (or moral), influenced by Confucianism or Buddhism, places an emphasis on reflecting ("hansei") on one's own mistakes and appreciating contributions from others, which encourages *cooperation* among children (cf. Lewis, 1995). Though "hanashi-ai" may eventually conclude which solution is better, correct, efficient, elegant, or whatever, competition among children is generally discouraged. Therefore, in principle, no winner and no loser exists in "hanashi-ai," unlike the Western-style argumentation.

*Proof*

No systematic research on relationship between the instruction of mathematical proof and Japanese communication styles has been conducted. The following argument is based on my own observation of Japanese classroom instruction of proof, analysis of textbooks, and my own cultural intuition as a Japanese. Therefore, this argument should be considered a working hypothesis for future study.

**Conception of proof.** Mathematics lessons in Japanese schools emphasize "wakaru" (understanding) of mathematical ideas. Memorizing formulas and mastering skills are not considered to be the central theme of learning. In school mathematics, teachers encourage children to understand reasons used in mathematics. Reasons are termed "wake" or "riyu." The activities of finding and explaining "wake" or "riyu" are considered essential for learning of mathematical proof in Japan.

Mathematical proof is called "shoumei" in Japanese. In junior high schools, explaining "wake" or "riyu" is often simply called "setsumei." Activities of doing "setsumei" are commonly held before introducing the concept of mathematical proof. The terms "wake," "riyu," and "setsumei" are all commonly used in students' everyday life. In contrast, the term "shoumei" rarely appears in everyday life; therefore, it has to be explicitly introduced and instructed in school. In Japanese schools, the term "shoumei" is first introduced to the students in geometry lessons of the eighth grade mathematics. Then, "shoumei" of a mathematical claim is usually defined as showing that the claim is true, using given conditions, and already accepted things as true. And, "shoumei" is conceived of as a special kind of "setsumei," characteristic of mathematics.

Table 2

*Consistency Between Doing Proof ("Shoumei") and the Group Model*

|  | "Shoumei" | Interpretation of "shoumei" | Group model |
|---|---|---|---|
| GOAL | "a valid conclusion" | a unanimous conclusion | social harmony ("wa") |
| SUBJECT MATTER | "wake" or " riyu" ( reasons why a claim is true) | loyalty to the things shared in the community | interpersonal attitudes (human relations) |
| MEANS | "given conditions," " already accepted things as true" | shared knowledge and expressions | intuition based on shared backgrounds |
| DIFFERENCES OF OPINION | not applicable | negligible (What is valid is not a matter of personal opinion) | to be avoided |

The instruction of mathematical proof seems to have been traditionally conceived of in the above mentioned group model of Japanese communication (see Table 2). The goal of proof is to reach a *unanimous* conclusion, which helps establishing the harmony in the community. Proof requires to *follow* the premises accepted in the community, which helps keeping the harmony of the community members. Proof relies on the things shared in the community: "Hanashi-ai" in the group model relies on the shared feelings and ideas in the community. Using those properties that have not yet been accepted ("proved") is prohibited. In proofs, what is valid is not a matter of

personal opinion; therefore, differences of opinion are not valued in proofs as in "hanashi-ai" in the group model. "Definitions" are used to clarify meanings of mathematical terms in order to avoid any confusion among community members.

Thus, this process of showing proofs seems to fit well the group model of Japanese public communication, having appeal for the cultural theme "wa" in Japan. The instruction of mathematical proof and the structure of "hanashi-ai" in Japanese classrooms seem more consistent with Japanese traditional communication styles than Toulmin model. The idea that learning of mathematical proof should be located in the context of argumentative activity seems highly biased from Western culture.

**Writing of proof.** Mathematical proving has a practice of producing *written* texts as a distinct aspect (Ernest, 1998, p.168). Communication models seem to shape writing styles of mathematical proof also.

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.
Given: $\angle B \cong \angle C$
Prove: $\overline{AB} \cong \overline{AC}$                              [ a diagram of a triangle ABC]
**Proof:**

| Statements | Reasons |
|---|---|
| 1. Draw the bisector of $\angle A$, intersecting $\overline{BC}$ at $D$ | 1. By the Protractor Postulate, an angle has exactly one bisector. |
| 2. $\angle BAD \cong \angle CAD$ | 2. Def. of angle bisector |
| 3. $\angle B \cong \angle C$ | 3. Given |
| 4. $\angle BDA \cong \angle CDA$ | 4. If two ∠s of one △ are $\cong$ to two ∠s of another △, then the third ∠s are $\cong$. |
| 5. $\overline{AD} \cong \overline{AD}$ | 5. Reflexive Property |
| 6. $\triangle BAD \cong \triangle CAD$ | 6. ASA Postulate |
| 7. $\overline{AB} \cong \overline{AC}$ | 7. Corresponding parts of $\cong$ △s are $\cong$. |

*Figure 3.* A two- column proof (Jurgensen et al., 1988, p. 125).

In the United States the two-column form is a common format for writing proofs in high school geometry (Figure 3), and sometimes in algebra also. In this format a proof looks like a spreadsheet calculation. One draws a diagram to illustrate the given information and lists in terms of the diagram the given information and the statement to prove. Then, one draws a long horizontal line and a vertical line downward from the middle, creating two columns under the horizontal line. In the left column, one writes a deductive sequence of statements leading to the statement to prove, numbering each statement. For each step of the deduction one has to write in the right column a reason for the deduction with a corresponding number. Because the format arranges a proof in two columns, it is called the two-column form in the textbook.

This format encourages explicit expression of reasoning. Every statement accompanies a reason slot, so it is hard to skip justifications without being noticed. In addition, all the postulates, definitions, and theorems necessary for justification are explicitly formulated in textbooks. This way of writing proof fits well again Western belief that

argumentation is highly valued.

On the other hand, proofs in geometry in Japanese junior high schools are usually written in "paragraph form," the writing style in ordinary life (Figure 4).

[In △ABC, if ∠B = ∠C, then AB = AC]
[Proof] Draw the bisector of ∠A. Let its intersection with BC be D.
In △ABD and △ACD,

$$\angle B = \angle C$$
$$\angle BAD = \angle CAD \ldots\ldots\ldots\ldots$$

Because the sum of the interior angles of a triangle is 180, the rest of the angles are equal,

$$\angle ADB = \angle ADC \ldots\ldots\ldots\ldots$$
$$AD \text{ is common} \ldots\ldots\ldots$$

Because, from , , and , one side and the angles at its both ends in each triangle are equal to the corresponding parts of the other, respectively,

$$\triangle ABD \equiv \triangle ACD$$
Therefore, AB = AC

*Figure 4.* A proof in a Japanese textbook (Tokyo Shoseki, 1997, p.118).

In this style, students do not have to write a reason for every statement. Also, not all postulates, definitions, and theorems involved with the proofs are formulated in textbooks. It is considered sufficient if a proof communicates its main ideas.

It would be very difficult for Japanese junior high school students to write reasons for every step. Even currently, it is well-known that mathematical proof is one of the most difficult topics in junior high school mathematics in Japan. The Japanese communication styles encourage students to far less express logical reasons than Western one as discussed earlier. The group model has provided little impetus for students to learn how to express their reasoning in formal writing.

## Concluding Remarks

This paper examined relationships between argumentation and mathematical proof from a cultural perspective, and concluded that the idea of locating learning of mathematical proof in the context of argumentation is biased by Western culture.

This conclusion suggests that it would be culturally difficult for Japanese to actualize the argumentation of Toulmin model in the classroom, and also to teach mathematical proof as argumentative activity. Actually, raising argumentation in Japanese classrooms is not easy generally, as Barnlund pointed out:

Communication in Western culture can be described as predominantly discursive and dialectic in form. It consists of exploring issues by alternately posing and answering a sequence of questions. The outcome depends on the relevance and clarity of the questions, and upon the accuracy and completeness of the answers.

The pattern appears in classrooms, courts of law, parliamentary bodies, deliberative committees, in ordinary conversations.

But it is not the dominant mode in Japan. Here direct questions are often regarded as a breach of manners. Outsiders teaching in Japan find that raising questions in classrooms fails to stimulate discussion ... (Barnlund, 1975, p.135)

Japanese society is moving towards internationalization. Communication and collaboration with people from different cultures have markedly increased. Young Japanese need to prepare for this internationalized society. Learning through the argumentative activity may not fit Japanese traditional patterns, but it would provide Japanese students with opportunities to acquire an important power for coping with the coming world. (For a rare attempt to use a debate between students in a Japanese junior high school geometry, see Ohta, 1995)

## Notes

## References

Balacheff, N. (1987). Processus de preuve et situations de validation. *Educational Studies in Mathematics*, *18*, 147-176.

Balacheff, N. (1991a). The benefits and limits of social interaction: The case of mathematical proof. In A. J. Bishop, S. Mellin-Olsen, & J. van Dormolen (Eds.), *Mathematical knowledge: Its growth through teaching* (pp. 175-192). Dordrecht, The Netherlands: Kluwer.

Balacheff, N. (1991b). Treatment of refutations: Aspects of the complexity of a constructivist approach to mathematics learning. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 89-110). Dordrecht, The Netherlands: Kluwer.

Balacheff, N. (1999, May/June). Is argumentation an obstacle? *International Newsletter on the Teaching and Learning of Mathematical Proof*. Retrieved from http://www-didactique.imag.fr/preuve/

Barnlund, D. C. (1975). *Public and private self in Japan and the United States: Communicative styles of two cultures*. Tokyo: The Simul Press.

Becker, J. P., Silver, E. A., Kantowski, M. G., Travers, K. J., & Wilson, J. W. (1990). Some observations of mathematics teaching in Japanese elementary and junior high schools. *Arithmetic Teacher*, *38*(2), 12-21.

Befu, H. (1980). The group model of Japanese society and an alternative. *Rice University Studies*, *66*(1), 169-187.

Boero, P. (1999, July/August). Argumentation and mathematical proof: A complex, productive, unavoidable relationship in mathematics and mathematics education. *International Newsletter on the Teaching and Learning of Mathematical Proof*. Retrieved from http://www-didactique.imag.fr/preuve/

D'Andrade, R. (1995). *The development of cognitive anthropology.* Cambridge, UK: Cambridge University Press.

Douek, N. (2000, August). *Comparing argumentation and proof in a mathematics education perspective.* Paper presented at the ninth International Congress on Mathematical Education, Topic Study Group 12 (Proof and Proving in Mathematics Education), Tokyo/Makuhari, Japan.

Duval, R. (1999, November/December). Questioning argumentation. *International Newsletter on the Teaching and Learning of Mathematical Proof*. Retrieved from http://www-didactique.imag.fr/preuve/

Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. Albany, NY: State University of New York Press.

Hanna, G. (1996). The ongoing value of proof. In M. de Villiers & F. Furinghetti (Eds.), *Proceeding of the eighth International Congress on Mathematical Education, Topic Group 8: Proofs and Proving: Why, when and how?* (pp. 1-14). Centrahil, South Africa: Association for Mathematics Education of South Africa.

Jurgensen, R. C., Brown, R. G. & Jurgensen, J. W. (1988). *Geometry*. Boston: Houghton Mifflin.

Kitade, R. (1993). [The human relation and communication in Japanese people]. In M. Hashimoto, & B. Ishii (Eds.), (pp. 23-54). Tokyo: Kirihara Shoten.

Lakatos, I. (1976). *Proofs and Refutations: The logic of mathematical discovery*. Cambridge, UK: Cambridge University Press.

Lakoff, G. & Johnson, M. (1980). *Metaphors we live by*. Chicago: The University of Chicago Press.

Lewis, C. C. (1995). *Educating hearts and minds: Reflections on Japanese preschool and elementary education*. Cambridge, UK: Cambridge University Press.

Moeran, B. (1984). Individual, group and *seishin*: Japan's internal cultural debate. *Man*, *19*, 252-266.

Nakayama, O. (1989). [Psychology of "ambiguous" attitude]. Osaka: Sogensha.

Ohta, S. (1995). Teaching figures by encouraging students to construct the world of geometry: Incorporating debates on "the sum of exterior angles of a concave quadrangle is 360 ." *Journal of Japan Society of Mathematical Education*, *77*(5), 11-19.

Shore, B. (1996). *Culture in mind.* New York: Oxford University Press.

Sekiguchi, Y. (1991). An investigation on proofs and refutations in the mathematics classroom. *Dissertation Abstracts International*, *52*, 835A-836A. (UMI No. 9124336)

Tokyo Shoseki (1997). [New mathematics 2: New edition]. Tokyo: Tokyo Shoseki.

Toulmin, E. S. (1958). *The uses of argument*. Cambridge. UK: Cambridge University Press.

Toulmin, E. S., Rieke, R., & Janik, A. (1984). *An introduction to reasoning* (2nd ed.). New York: Macmillan.