MATHEMATICS LESSONS IN KOREA:
TEACHING WITH SYSTEMATIC VARIATION

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To investigate the features of Korean mathematics lessons, the data from the Learners' Perspective Study (LPS) was analyzed. A cursory review of the LPS data gives the impression of a very traditional style of teaching, a salient feature of which is the dominance of teacher talk and reticence of students. Instruction seemed to focus more on the content rather than the process of mathematics, with concepts often stated directly using formal mathematical language.

In a more fine-grained analysis, one lesson judged to be "typical" in each of the three schools was selected for study based on the 'theory of variation.' The results show that there were rich variations in concepts, procedures, and practicing exercise. In particular, a kind of systematic and continuous variations is identified. These variations started with a certain simple basic situation, and one aspect of the situation was then varied at a time until a target form was reached. It is argued that these systematic variations constitute a kind of exploration on the part of the students. These variations were carefully designed by the teacher, leading students to discern a certain set of attributes of the concepts involving the final situation. Coupled with systematic variations in the exercise given to students where they have an opportunity to practice the application of the concepts systematically in class and/or at home, such systematic variations will create the necessary condition for critical attributes of the object of learning to be experienced by the students.

Introduction

In the past decade or so, there has been increasing interest in the study of mathematics classrooms in East Asian countries, or countries falling under the so-called Confucian-heritage culture (CHC), the dominant culture in East Asia. However, relatively little has been published in the international literature on classroom practices in the CHC country of Korea. In this paper, characteristics of the Korean mathematics classroom that are deemed to be conducive to effective learning are identified through an analysis of the Korean data of the Learners' Perspective Study (LPS). Then the classroom characteristics identified are interpreted in terms of the underlying cultural values that they share with other East Asian countries.

Learners' Perspective Study

The LPS, a video study of the mathematics classroom, is characterized by in-depth documentation of the student perspective over several lessons in the same classroom. The methodology of the LPS offers an informative complement to the survey-style approach of the TIMSS video study. A research design of LPS predicated on a nationally representative sampling of individual lessons, as in TIMSS, inevitably reports a statistically-based characterization of the 'typical lesson'.

In the LPS, one teacher from each of three schools in each participating country was sampled for study, and a series of 10 to 15 consecutive lessons taught by the teacher were videotaped. The teachers chosen were judged to be competent teachers in their respective countries. The study combines videotape data with participants' re-constructions of classroom events. Three cameras were employed in the videotaping; a "Teacher Camera," a "Student Camera" and a "Whole Class Camera." An audio-video mixer was used for on-site mixing of the images from the teacher camera and the student camera to provide a split-screen record of both teacher and student actions. The integrated images were used for stimulated recall in interviews conducted immediately after the lessons to get students' reconstructive account of the teaching and learning (Clarke, 2004).

Theory of variation[1]

To identify mathematics classroom features, a learning theory espoused by Marton (1999) is utilized in the analysis of the Korean data. Marton hypothesized that variation, simultaneity, and discernment were critical to learning, and studies by Runesson (1999) and Mok (2000) showed that Marton's theory of variation had a demonstrated potential in revealing the salient characteristics of classroom features that are related to student learning.

The theory of variation was developed from the work of Marton and Booth (1997), which described how an 'enacted space of learning' was constructed through the creation of certain dimensions of variation for the experience of the students. According to Marton *et al* (2003), learning is a process in which learners develop a certain capability or a certain way of seeing or experiencing. In order to see something in a certain way the learner must discern certain features of the object. Experiencing variation is essential for discernment, and is thus significant for learning, and Marton *et al* (2003) argued that it is important to attend to what varies and what is invariant in a learning situation.

[1] Theory of variation is based on Phenomenography, which was developed by a Swedish research group in early 1970s. The word 'phenomenography', coined by Marton in 1979, was derived from the Greek words 'phainemenon' and 'graphein', which mean appearance and description respectively. Thus 'phenomenography' concerns about the description of things as they appear to us. According to phenomenography, a way of experiencing something is defined in terms of the critical aspects of the phenomenon as discerned and focused upon by the experiencer at the same time. Nobody can discern an aspect of a phenomenon without experiencing variation in a dimension which corresponds to that aspect (Marton & Booth, 1997; Pang, 2003). This provides a basis for the theory of variation.

In parallel with Marton's theory of variation, a theory of mathematics teaching and learning, called teaching with variation, has been developed by Gu (1994). Gu's theory was based on a series of longitudinal mathematics teaching experiments in China, and was heavily influenced by theories of cognitive science and constructivism. According to this theory, meaningful learning enables learners to establish a substantial and non-arbitrary connection between their new knowledge and their previous knowledge (Ausubel, 1968). Classroom activities can be developed to help students establish this kind of connection by experiencing certain dimensions of variation. The theory suggests that two types of variation are helpful for meaningful learning. One is called "conceptual variation", and the other is called "procedural variation" (Gu *et al*, 2004).

Conceptual variation consists of two parts. One part is composed of varying the connotation of a concept: standard variation and non-standard variation. The other part consists of highlighting the substantial features of the concept by contrasting with counterexamples or non-examples. The function of this variation is to provide learners with multiple experiences from different perspectives.

Procedural variation is concerned with the process of forming a concept logically and/or chronologically (scaffolding, transformation), arriving at solutions to problems, and forming knowledge structure (relationship among different concepts). The function of procedural variation is to help learners acquire knowledge step by step, develop learners' experience in problem solving progressively, and form well-structured knowledge.

Multi-dimensional Variation and Developmental Variation

While the two kinds of variations suggested by Gu are potentially powerful tools for analyzing classroom events, the terms "conceptual variation" and "procedural variation" may be misleading. The adjectives "conceptual" and "procedural" may remind readers of the terminology of "conceptual understanding" and "procedural understanding" coined by Hiebert (1986), which are used differently from the meaning of "conceptual" and "procedural" as defined by Gu. Gu's terminology may give the impression that "conceptual variation" and "procedural variation" are disjoint, but in fact according to Gu's own definition, procedural variation is also related to the formation of concept. So the terms "conceptual variation" and "procedural variation" do not reflect very well the meaning they are supposed to represent as defined by Gu.

In this paper, the term "multi-dimensional variation" will be used to denote what Gu termed "conceptual variation" because the term refers to enhancing conceptual understanding through multiple representation and varied examples of a given concept. Along with conceptual variation, the term "developmental variation" will be used to substitute for Gu's "procedural variation", since this variation helps the learners to construct knowledge structures through progressively acquiring the knowledge.

For example, in one of the lessons videotaped, the teacher familiarized students with the concept of linear equations in two unknowns through comparison with linear equations in one unknown. The teacher reminded the class that equations with one unknown and those with two unknowns are similar in the sense that a root should satisfy the equation when substituted into the unknown(s) of the equation. But the two are different because the number of roots is different. This explanation helps students to understand linear equations with one unknown and two unknowns by contrasting the similarities and differences of the two concepts, and is thus considered a "multi-dimensional variation."

Another example of multi-dimensional variation is found in a lesson from another school. There the teacher introduced a new concept (ratio of areas) through concrete examples in everyday life: the fact that the amount of ink needed to print a photo depends on the area of the photo. This connection between an abstract mathematical concept and a concrete example in real life can be interpreted as a multi-dimensional variation as well as "mathematization" in Freudenthal's terms (Freudenthal, 1983).

An example of "developmental variation" is identified in a lesson from the third school videotaped in this study. In the lesson, the teacher provided a variety of situations by presenting a pouch with colored stones and then changing a certain colored stone to another colored stone. Based on this "experiment," students observed that the probability increased from 0 progressively until eventually it reached 1. This was then generalized into the properties of probability. So students acquired the knowledge through experiencing progressive problem solving.

In fact, these notions of variations are similar to the "mathematical variability principle" by Dienes (1973), and the "duality of mathematical concept" suggested by Sfard (1991). According to this theory of variation, the "space of variation" consists of different dimensions of variation in the classroom, and they form the necessary condition for students' learning in relation to certain learning objectives. For the teacher, it is crucial to consider how to create a proper space of variation focusing on critical aspects of the learning object through appropriate activities. For the learner, it is important to experience the space of variation through participating in constituting the space of variation.

For the data analysis in this paper, the patterns of variation critical to learning will be described in two aspects: what the multi-dimensional and developmental variations are and how they are created. Studies by Runesson (1999) and Rovio-Johansson (1999) support the hypothesis made by Marton (2000) that variation is a key for comparing the difference in practices between the East and the West. Marton argued that the most important difference between the Chinese/Japanese classes and those in the U.S. was the difference in the pattern of variation. Chinese and Japanese students learned to approach the same mathematics problem in different ways, whereas the

American students learned to apply the same approach to different but similar problems.

Sample, data collection and analysis

Following the methodology of LPS, three schools in the urban/metropolitan community of Seoul were sampled for study. To preserve anonymity, the three schools are referred to as school H, school K and school W in this chapter. One grade 8 mathematics teacher in each of schools H, K and W judged to be competent by the local professional community was selected. The teacher had at least five years of experience as a qualified teacher. One of the grade 8 classes taught by the teacher was then selected for study, and a continuous sequence of at least 10 lessons were videotaped for the class.

The videotaped lessons were then viewed carefully, and a preliminary analysis was preformed on the data. Then a lesson in each of the three schools judged to be "typical" of lessons in the series was chosen for a more fine-grained analysis. Table 1 shows the background characteristics of the three sampled schools and the sampled teachers, as well as information about the lessons chosen for detailed analysis:

Table 1. Background characteristics of the sampled schools and lessons

	School H	School K	School W
Type of schools	Girls' school	Co-educational	Co-educational
SES of parents	Mostly middle class		
Teacher Gender (age)	Male (47)	Female (32)	Female (33)
Teaching Experience	18 years	6 years	7 years
Class size	36	34	37
Duration of lesson	45 minutes		
Topic of lesson	Linear equations with two unknowns	Area of similar geometric figures	Properties of probability

Results

Preliminary Analysis

A preliminary analysis of all the videotaped lessons shows that the Korean classrooms in this study, like the classrooms in other East Asian countries, were characterized by the dominance of teacher talk and reticence of students when compared with Western countries (Leung and Park, 2005). The number of words spoken by the teachers and the students in all the lessons in the three schools was counted and their ratio computed, and the results are shown in Figure 1. As can be seen from Figure 1, the ratios of number of words spoken by the teacher to those spoken by the students vary between 18 and 40, with an average of 28. These ratios are higher than those obtained from the TIMSS 1999 Video Study (Hiebert et al, 2003), especially higher than those for the Western countries in the TIMSS 1999 Video Study (Figure 2)

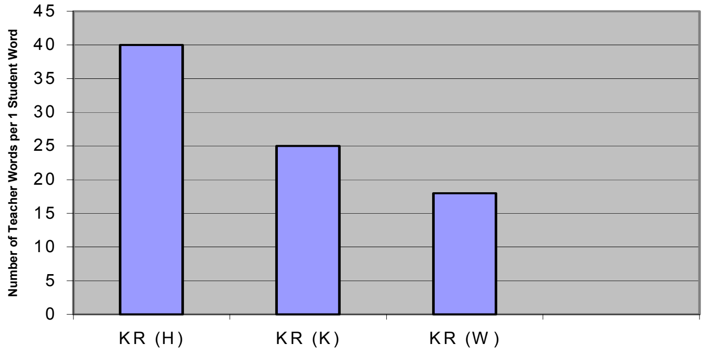


Figure 1. Ratio of number of teacher words to student words in the 3 Korean schools

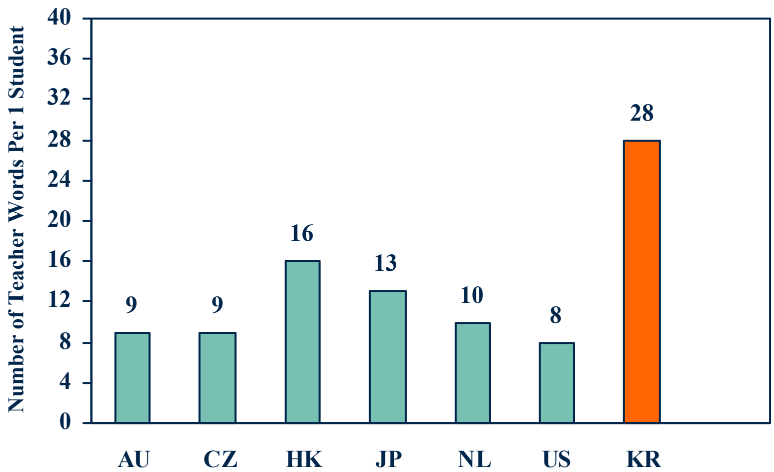


Figure 2. Ratio of number of teacher words to student words in Korean classrooms compared to those in other countries

A cursory review of the video data shows that the teaching in the three Korean schools seemed to focus more on the mathematics content to be learned rather than the process of understanding the content. Mathematics content was delivered efficiently, with mathematics concepts often stated directly. As the teacher in school H remarked during the interview after the lessons:

Teacher of school H:

Of course, there should be lots of student activities. But I found that they distracted the students and made it difficult to proceed with the lesson. Also, the high achieving students seemed to get bored and would sometimes just sit idled. If we have activities in class, those who are not so good don't even know what they are for. Innovative lessons which try new thing in class make everybody tired. Just giving mathematical explanations is much better for both high and low achieving students. People seem to think that inquiry instruction is a good form of teaching that fits the current trend but I do my own explanation and lead the whole class because it (inquiry instruction) tends to loosen the lesson somewhat.

The focus of the lessons seemed to be on the final product rather than the process of arriving at the product. There was much more use of formal mathematical language rather than less formal everyday life language such as metaphors. There was also ample practicing of mathematics exercise during the lessons.

The analysis also shows that the Korean lessons by and large followed a rather similar structure, which we categorize into four stages. In the first stage, which we name *review and induction*, the teacher would usually begin the lesson by reviewing relevant materials covered in previous lessons and prepare the way for the main concepts of the lesson to be introduced. In the second stage, named *exploring new concepts*, the main concepts of the lesson would be introduced and elaborated by teacher-initiated exploration. In the third stage, *examples and exercise*, the main concepts would be illustrated with examples, and students would be directed to work on some relevant exercise. In the final stage, *summary and assignment*, the teacher would summarize the main points of the lessons and assign homework for the lesson.

Fine Grained Analysis

As pointed out above, the preliminary data analysis was followed by a more fine-grained analysis of one lesson in each of the three schools judged to be "typical" of lessons in the school. The further analysis was data-driven, following a grounded theory approach. The lessons were reviewed several times and the variations were identified from the process. Results of this analysis of the three chosen lessons show that during the four stages of the lesson identified above, there were a lot of variations in concepts and practicing exercise. In the discussion below, we denote the variations referred to by two capital letters and a number. The first letter refers to the school (H, K or W) where the lesson took place, the second letter stands for either multi-dimensional variation (M) or developmental variation (D), and the number indicates the order in which the variation occurred in that particular lesson. For example, HD1 means the first developmental variation which occurred in the chosen lesson of school H.

The multi-dimensional and developmental variations identified in the three lessons include:

–Linkage of different concepts, introducing a new topic based on a review of the content covered in previous lessons (HD1 and WD1)

–Consolidation through summary (HM4)

–Learning concepts through comparison and contrast (HM1)

–Linkage between mathematics and concrete examples (KM1)

–Multiple representation of a concept (HM2, KM2)

–Generalization through abstraction (WD2)

Systematic Variation

One particular kind of variation warrants highlighting for discussion. It is a kind of systematic and continuous variation that leads students to understand the concept under discussion. It is interesting to find that such systematic variations were found in each of the three lessons analyzed: HM3 & HM4, KM3 & KM4, and WM1 below can all be classified as this kind of systematic variation.

HM3 & HM4

This is the first lesson of school H. Students were given a series of tasks which were gradual variations to a basic equation, $x + y = 5$.

	linear equation with 2 unknowns	domain of x and y
basic equation	$x + y = 5$	natural numbers
	⇩	
task 1	$3x + y = 15$	natural numbers
	⇩	
task 2	$-3x + y = 12$	natural numbers
	⇩	
task 3	$x + 2y = 6$	x values {**-2, -1, 0**, 1, 2, 3, 4, 5, 6, 7}
	⇩	
task 4	$x + y = 3$	no limitation

In task 1, the domain of the unknowns was natural numbers and only the coefficients were changed from the basic equation. In task 2, the equation included a negative coefficient, and in task 3, the domain of the unknowns was extended to negative integers. In task 4, the domain of the unknowns was further extended to real numbers, and students were required to draw a graph without going through the process of finding the solution. After solving task 4, the concept that the graph of a linear equation with two unknowns was a straight line in the coordinate plane was well expounded (HM3).

1. T: Let's go ahead and read what's written right below. There is only one line between two points. Therefore, to draw the graph of a linear equation, one would get two roots and graph those two points on the coordinate plane. With those two points, one may easily construct a line.
2. T: So, if we are to draw a line on the coordinate plane we would need to choose two ordered pairs and connect them. What do we get then? We get the graph of a line. There is only one line when we have two points, right? When we connect our points, we get a line.

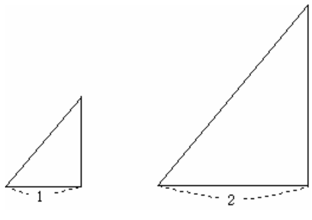
At the end of the lesson, the contents covered thus far were summarized. During this process, the general form of a linear equation with two unknowns ($ax + by + c = 0$) where the domains for the unknowns (x and y) are real numbers was finally introduced. This is a kind of multi-dimensional variation to consolidate and enhance the formation of concepts (HM4).

In the analysis above, we can see that there are systematic variations starting with the basic equation $x + y = 5$ and moving step by step to the general form of $ax + by + c = 0$. In each variation, all but one of the components of the equation concerned are kept constant, so that the effect of the varied component is elucidated.

KM3 & KM4

This is the seventh lesson of school K. Students were given tasks which were variations of a basic diagram. Tasks 1, 2 and 3 were not that different from the basic diagram because students were required to find the ratio of areas when only the type of geometric figure and ratio of similarity were different. Task 4 required students to generalize from what they discovered in the preceding tasks. Here, task 1 serves as scaffolding for tasks 2 and 3 as well as task 4. (KM3)

Task1: Find the ratio of similarity and the ratio of areas of the given figures.



Task2: Find the ratio of areas of the rectangles when the ratio of sides is 1:3.

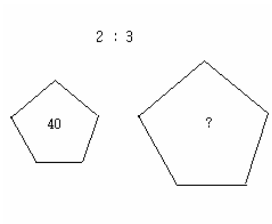
Task3: Find the ratio of areas of the triangles when the ratio of sides is 2:3.

Task4: Fill in the blanks:

The ratio of areas between similar figures is the ____ of the ratio of sides.

The ratio of areas between similar figures is ____ when the ratio of sides is m:n.

Task5: Compute the area of a large pentagon when the ratio of similarity between two pentagons is 2:3 and the area of the small pentagon is 40.



	geometric figure	ratio of similarity	find
basic diagram	rectangle	1:2	ratio of the areas
task 1	triangle	1:2	ratio of the areas
task 2	rectangle	1:3	ratio of the areas
task 3	triangle	2:3	ratio of the areas
task 4	arbitrary polygon	m:n	ratio of the areas
task 5	pentagon	2:3	the area of one pentagon

The lesson proceeded gradually from the basic diagram through tasks 1 to 4, but task 5, which was given as a review problem of the lesson, made a greater variation to the original problem compared to the preceding tasks (KM4). The geometric figure is not the familiar triangle or rectangle but a pentagon (for which the area is not easily found), and the problem is not to find the ratio of areas but to find the area of one pentagon based on the area of another.

With these systematic variations, students are guided to understand the concept that for a pair of *any* similar polygons, if the ratio of similarity is m : n, then the ratio of areas is $m^2 : n^2$. Students are expected to be able to find the area of a polygon based on the area of another similar polygon and the ratio of similarity.

WM1

This is the first lesson of school W. The first content to be covered in the lesson was that the probability of an impossible event is 0, that of a certain event is 1, and all probabilities have values between 0 and 1. The teacher did not present the problem in heterogeneous situations but found the probability of a series of situations by continuously changing the color of stones in the same pouch. The teacher drew a pouch on the board, stuck three red magnets in the pouch and showed the students that the probability of selecting a blue stone is 0. Then she replaced one red magnet for a blue one and showed that the probability of choosing a blue stone was then 1/3. By replacing a red magnet by a blue magnet one by one, the class eventually found that the probability of selecting a blue stone when all three stones are blue becomes 1.

In the three examples above, the teaching all started with a certain simple basic equation, diagram or situation. Then only one of the different aspects of the basic equation, diagram or situation was varied at a time, and the variations followed a systematic pattern until the equation, diagram or situation reached a target form. It can be argued that these systematic variations constitute a kind of teacher-initiated exploration or guided ***exploration*** on the part of the students. It seems that the incremental variations were carefully designed by the teacher, leading students to discern attributes of the object of learning or the concepts involving the final situation.

In addition, in all three lessons analyzed above, there were also systematic variations in the exercise given to students. So students after being exposed to systemic variations in the presentation of the concepts would now have an opportunity to practice the application of the concepts systematically in class and/or at home. According to the theory of variation, these combined experiences of the students on the systematic variations of the concepts will help establish their understanding.

Discussion

It was mentioned above that the systematic variations identified constitute a kind of *exploration* on the part of the students. Exploration in the Western context often means students were given open-ended tasks and engaged in free exploratory activities, usually conducted in a small group or individualized setting. This is in contrast to the teacher-directed Korean classroom reported above. However, the fine-grained analysis of the data shows that in the seemingly teacher-directed Korean classroom, students still had the opportunity of exploring mathematics ideas under the close guidance of the teacher. In the words of the variation theorists, such systematic variations will create the necessary condition for different features or critical attributes of the object of learning to be experienced by the students (Marton and Booth, 1997). In this regard, this kind of exploration is referred to as teacher directed exploration or simply directed exploration in this paper.

The descriptions above fit well with the findings of another study on the classroom practices in Hong Kong and Shanghai. Huang and Leung (2004) reported that the Hong Kong and Shanghai mathematics classrooms in their study were characterized by teacher dominance and student active engagement, with much emphasis on exploration of mathematics and exercises with variation.

The East Asian Culture

How do we account for the classroom practices in Korea as identified in this study? To what extent can these classroom characteristics be attributed to the underlying East Asian culture? In the literature, various scholars have tried to attribute differences in classroom practices and achievements to cultural factors (Watkins and Biggs, 1996; Wong, 1998). In particular, Leung (1999) discussed the traditional Chinese views of mathematics and education which might have an impact on the classroom practices in the current Chinese classroom. Leung (2001) extended the argument from the Chinese classroom to the East Asian classroom and identified features of East Asian mathematics education in contrast to features in the West, and presented the differences in terms of six dichotomies. He argued that the different practices between East Asian classrooms and those in the West are based on different deep-rooted cultural values and paradigms, whether explicit or implicit, that have been built up over centuries. In the next section, we will try to account for some of the classroom practices identified in this study through referring to the underlying cultural values that Korea shares with other East Asian countries.

Teacher dominance and whole class teaching

Teacher dominance and whole-class teaching accord well with the traditional East Asian philosophy which emphasizes integration and harmony (Sun, 1983), in contrast to the Western culture which stresses independence and individualism (Taylor, 1987). Related to this tendency in the East Asian culture, which Yang (1981) labeled as ‘social orientation’ (as opposed to ‘individual orientation’), are characteristics such as compliance, obedience, respect for superiors and filial piety (Lin, 1988, Liu, 1986). East Asians are known to have a tendency of complying with rules or orders more than Westerners, giving rise to a strong tendency for uniformity and conformity (Bond and Hwang, 1986). In such a cultural environment, it is not surprising that classrooms are found to be teacher dominated, with whole-class teaching being commonplace.

Teacher dominance may also be related to the high regard given to teachers in the East Asian culture. In the East Asian culture, the image of the teacher is that of a scholar held with high respect. So it is just natural that in the classroom setting, teaching and learning activities should be directed by the scholar-teacher. Teacher dominance and whole-class teaching however do not necessarily mean that students are not actively engaged in the lesson. As can be seen from the results of this study presented above, active student engagement is still possible in a classroom where the class size is large and the activities are dominated by the teacher.

Content versus process

It has been reported in the literature that "Chinese teachers held the more rigid view of mathematics being more a product than a process, (and) the more important thing for them in mathematics teaching was to have the mathematics content expounded clearly." (Leung, 1995: 315). The emphasis in the East Asian mathematics classroom was on the mathematics content and the procedures or skills in dealing with the content rather than the process of handling mathematics. There is an underlying belief that

> "the critical attribute of mathematics is its distinctive knowledge structure, and it is this distinctive structure which distinguishes mathematics from other forms of knowledge. So the most important goal of mathematics learning is to understand and get hold of this distinctive knowledge structure, and the foremost task of the mathematics teacher is to help students acquire the mathematics content. The process of doing mathematics is part of the process of learning the content, but the process needs the content as its foundation. Without content, there is nothing for the process to be applied to"
>
> (Leung 2001: 39)

The findings of this study agree well with the reported views above. This stronger stress on the content rather than the process of mathematics also reflects how the nature of mathematics is perceived in the East Asian culture.

The emphasis on directed exploration and practice

The finding in this study on the emphasis of the Korean teaching on directed exploration may seem to contradict the stereotype of the East Asian classroom. The learning styles in East Asia are often portrayed in the literature as "learning by rote" or "passive learning" (Biggs and Watkins, 1996), and the teaching strategies characterized as "procedural" (Zhang, Li and Li, 2003). But results of this study show that behind the seemingly procedural teaching and passive learning, the Korean students are actually heavily involved in exploration when following the prescribed classroom activities designed by the teacher.

On the other hand, the finding that there are a lot of practicing exercises in the Korean lessons is consistent with the stereotype many held for the East Asian classroom. However, the results of this study also suggest that the exercises that Korean students worked on were not simply repetitive drills, but were carefully designed problems with systematic variations.

In the East Asian culture, practice has always played an important role in the learning process. Actually, the word or term in Chinese for “learning” consists of two characters (学习), and the second character (习) conveys the meaning of practice. So in the CHC tradition, practice is an inherent part of the learning process. The idea of learning without practicing is absurd in the CHC. The well known saying (熟能生巧) which is often translated as “practice makes perfect”, reflects this philosophy of learning well. As Confucius put it, “Is it not a pleasure, having learned something, to try it out (i.e., practice) at due intervals?” (*Analects,* I. 1).

Underlying this stress on practice are the traditional East Asian cultural values which lay a strong emphasis on the importance of education and which attribute achievement more to effort than to innate ability. Under the influence of such values, education or study is considered a serious endeavor, and there is a high expectation for students to put in hard work and perseverance in their study and to achieve. This is reinforced by a long and strong tradition of public examination, which acts as a further source of motivation for learning. All these add up to form an important source of motivation for students to learn well and to excel.

Conclusion

As can be seen from what have been presented in this chapter, the analysis of the Korean LPS data utilizing the theory of variation has yielded some interesting results which help reveal the kind of teaching in Korea. Ample practice of mathematics skills does not necessarily imply rote learning or learning without understanding. The analysis in this study shows that there are actually well designed and systematic variations in both the classroom activities and the practicing exercises in the Korean classroom, with the consequence that a lot of exploration is taking place on the part of the students in the teacher-directed classroom. And according to the theory of variation, such experience of variations on the part of the student will lead to understanding. As Leung (2001) pointed out, understanding is “not a yes or no matter, but a continuous process or a continuum.” The process of learning often starts with gaining competence in the procedure, and then through “continuous practice with increasing variation” (Marton, 1997), students gradually gain understanding.

Amidst the global tide of educational reform, there is a pressure on governments of East Asian countries, Korea included, to change the educational practices in their countries as well, and a common strategy taken is to send a team of policy makers to a number of “more developed” countries and shop around for new ideas and practices. But too often, those new ideas and practices have not been well tested even in those “developed” countries, and the cultural differences between the East Asian countries and the “developed” countries being visited have not been attended to in the adoption of the reforms. What is needed in Korea and other East Asian countries for policy decision are systematic collection and analysis of relevant data, and reflection on the strengths and weaknesses of the existing system and the interaction between existing educational practices and the underlying culture. And what is reported in this paper represents exactly one such endeavor.

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