

BEGINNING THE STUDY OF THE ADDITIVE FIELD

Grecia Gálvez Pérez

Ministry of Education. Chile.

An innovation project is developing in Chile, the Strategy of Consultanship to the School for the Curricular Implementation in Mathematics, that considers the mathematical activity as the study of problems fields. In this article a first grade class is described and analysed, carried out by a "consulting teacher" from this Strategy who is responsible for the training of other teachers. The training is carried out through the study and application of Didactic Units that are proposals to organize brief learning processes in those the sequence of the proposed tasks generates the evolution of the techniques and knowledge that the students put at stake.

THE STRATEGY OF CONSULTANTSHIP TO THE SCHOOL

The level of Basic Education includes eight years in Chile, and it has a covering about 97%. About 92.8% of the children study in schools subsidized by the State, administered by a Town council (52.3% of the registration) or by a private supporter (40.5% of the registration). The Ministry of Education regulates the curriculum, evaluates the students' performance and carries out diverse initiatives to improve the educational system, such as training and evaluation for teachers, distribution of study texts and other resources for the learning, and implementation of special programs for schools with poor academic performance.

When pupils finish their elementary school 4th grade (ten year-old children) a performance measurement for all students is carried out. Evaluating the result of these measurements, and considering the new existing Study Programs from 2002, the Ministry of Education began a Reading-Writing-Mathematics Campaign (LEM), dedicated to improve pupils' elementary learning in the first school cycle. According to this Campaign, an agreement was settled down in 2003 between the Ministry and the Chilean University of Santiago to develop a "Strategy of Consultanship to the School for the Curricular Implementation in Mathematics". During the first year, a pilot plan was carried out in twenty schools and, during the two following years, this strategy was applied in almost two hundred schools, in three Regions of the country.

The Strategy of Consultanship to the School elaborated a didactic proposal being based on the Anthropological Theory of Didactics (Chevallard, 1999) that takes mathematical activity as an activity of fields' study of mathematical problems:

The mathematician does not only aspire to think about good problems and solve them, but rather he also seeks to characterize, to define and even to classify the problems in "types of tasks" to understand, describe and characterize the techniques that he uses to solve them, until the point of controlling them and regulating their use, he intends to settle down the conditions under which these are working or they are not applicable

and, ultimately, he aspires to build solid and effective arguments that sustain the validity of his ways of proceeding (Bosch, M., L. Espinoza y J. Gascon, 2003).

According to this theory, so that children learn mathematics it is necessary that they deal with problems, elaborate procedures to solve it, explain and justify the operation of their procedures, students exchange and compare procedures between themselves, they are willing to adopt those that are more effective to solve the outlined problem, explain the knowledge that support them and relate them with knowledge that already have, then they will deal a new problem located in the same field that presents new challenges for the students.

According to the Theory of the Didactic Situations (Brousseau, 1990), in this proposal is considered that the "sense" of a mathematical knowledge is built facing with a set of problematic situations where this particular mathematical knowledge appears like a tool for their solution. These situations should allow children to elaborate strategies from previous mistakes, from the inadequacy or "fail" from their previous knowledge and the modification of the same ones.

The Strategy of Consultantship to the School elaborated a model to transform the pedagogic practices, based on sixteen Didactic Units, for the first four elementary grades. Under a consulting teacher leading, the teachers of each school study these Didactic Units and afterwards they apply them, with the observation and feedback of the consulting teacher.

The class that I will present was carried out in 2005, in a council school of a town located 500 km to the south of the country's capital. The students are considered vulnerable or with social risk factors, due to its economic lacks and incidence of activities of socially disintegrating character. The teacher who imparts the class is a "consulting teacher" since 2004. She is recognized as a good teacher by the local ministerial authorities; she has been worthy of a special salary assignment for her "pedagogic excellence" and she has been selected to evaluate other teachers through a mechanism of "elaboration of briefcases". She is also recognized by the community, since parents of their current students accepted they were changed from afternoon to morning period, so that she was the teacher of their children.

The elected class corresponds to the first class of the Second Didactic Unit for the first elementary grade, elaborated by the Consultantship Strategy team. The Unit is titled: "Additive Problems of Composition" and it is a proposal to organize the study of this topic during three classes, from 90 minutes each one. The Unit contains an outline that allows to visualize the learnings that it is awaited students achieve, the learnings that should have previously acquired, the progression of proposed tasks and the awaited procedures during these three classes, besides plans for the classes that describe the activities to carry out, working sheets for students and an evaluation instrument¹.

¹ See in Appendix: Outline of the Didactic Unit, plan for the first class and work sheets for the students first class.

Description of a good class

The sequence of activities of the class gets close to the plan elaborated by the Strategy of Consultantship to the School, as introduction to the study of the additive problems. For a self-controlling mechanism, the teacher has written the activities in cards that she reads aloud simulating they are Pepito's letters, a pelican cut off in bristol board and superimposed at the corner of the blackboard. Following the plan, the class consists of three well differentiated moments, an *initial moment*, where children exercise abilities considered as previous to the addition and subtraction learning, a *development moment*, in which they carry out and observe actions for joining and separating collections of objects, associating them to the addition and subtraction operations, and a *closing moment*, in where what was learned is institutionalized.

In the *initial moment*, the plan proposes:

- Say the numeric sequence in upward and descending form, at least up to 10. The teacher structures this activity forming a row of seven children and requesting them that they bend over; each one rises when saying a term of the sequence in upward order, and they bend over when saying it in descending order again. According to children's wish, the teacher accepts they also say the sequence from 0 to 6 and from 6 to 0.
- Count at least up to 10 objects. A Pepito's letter notices: "Children have not been counted". The teacher designates three children so that they count the members of their row. Numbers are registered on the blackboard, each of them digits. At Pepito's request, absent children are counted. The teacher reads their names and children put a stick on their tables for each name. The total sticks are 12.
- Read the numbers up to 10, and copy them from the numbered ribbon. At the beginning of the class two children order the first 25 numbers, written in cards with a rope. Once reordered, they read in a chorus. After counting the absentees, they identify the 12 in the rope, as well as the previous number and the following one. The teacher writes two digit numbers on the blackboard and children read them mentioning which it is bigger. At the initial moment children don't write numbers.

At the *moment of the class development*, the *mathematical task* specified in the plan is: "Determine the quantity of objects resulting from joining or separating actions, proposed verbally by the teacher, manipulating objects that are accesible to children. Numeric range up to 10."

According to the plan, the teacher organizes an activity in which children manipulate objects. In order to make activity more attractive, she includes a song about a hen, known by children. When saying each number, children put a stick on their table. The stories are:

- The hen put 3 eggs during the first day and 5 eggs the second. How many it put in the two days?

- Considering the 8 eggs it put, 3 were broken. How many are now?

In order to determine their answer, children count the objects that have on their table, awaited *technique* for this first class, according to the plan.

The teacher organizes another activity changing the *conditions of carrying out the task*. This time, the objects that join and separate -red and blue notebooks- they are only manipulated by the teacher. The children follow the actions visually and they determine, counting at distance, how many are they:

- 2 red notebooks plus 3 blue notebooks
- 3 red notebooks plus 3 blue notebooks
- 6 notebooks minus 3 blue notebooks (the same ones that were joined)
- 4 red notebooks plus 3 blue notebooks
- 7 notebooks minus 3 blue notebooks (the same ones that were joined)

In these exercises, the subtraction appears as inverse operation from addition, in which teacher enhances the proposal of the plan.

The plan intends to study the case of additions and subtractions in those the second term is 1. The teacher comments that they already know it, talking to Pepito. She continues manipulating notebooks:

- 4 red notebooks plus 1 blue notebook
- 5 notebooks minus 1 blue (the same one that was joined)
- 6 red plus 1 blue
- 7 notebooks minus 1 blue (the same one that was joined)

As last activity of this moment, the plan proposes for children 4 work sheets, with collections of drawn objects that can be separated in two subcollections. Children should count the objects in one or both subcollections and all the objects, sometimes answering in oral form and other in written form. In the first one, second and fourth work sheet, the objects are children or animals. In the third work sheet the objects are triangles and rectangles. The teacher draws an example of each one on the blackboard and she asks the children to identify them. When she reads that they should count the small rectangular figures, she comments: how difficult it is!

At the *closing moment*, the plan of the class indicates that the teacher should systematize what they learned, associating the action of joining collections with addition and the action of separating collections with subtraction. In order to determine the sum or the subtraction, the counting is used, as procedure. When one of the collections has only an object, the result is the following number, or the previous one. The teacher asks what they learned today. A girl says: I learned to subtract and to add and to do mathematics with the numbers. Another: And to count, with the sticks. Another: order the numbers, too. The teacher asks them to sing the Pepito's song, as they always finish the class, changing the letter to say what they learned today.

Why do I consider that this is a good class?

First of all, the structure of the class is consistent, it is adjusted to a proposal designed by a project specialized in the implementation of the national curriculum. The teacher segments the class in the three moments indicated in the plan and she is guided by the appropriate sequence of activities.

In relation to emotional environment, the teacher has a very significant support for children: Pepito, the clipped pelican. When feigning to dialogue with this character, the teacher introduces in the class an imaginative and funny dimension, full of surprises. It is necessary to order the numbers that Pepito disordered; it is necessary to sing "The Francolina hen" because Pepito likes it (and children too); it is necessary to sing the song dedicated to the pelican, to explain him what they learned in the class. This funny tone proposes the development of a positive attitude toward mathematics' learning.

The class is focalized in the mathematical task. The teacher introduces humorous comments and she accepts those from children, but she quickly recaptures the course of the work, giving clear signs of what it is the important in order to achieve the learning.

The work rhythm is intense; hard-working time. The course of the activities is continuous; practically there are not interruptions. Occasionally an adult enters and goes out discreetly, without distracting the group. The teacher has a voluntary assistant, the mother of a student, who distributes materials and assists some children with basic necessities. In certain moment, a girl comes closer to the teacher for asking sticks; the teacher is looking for the sticks and then gives her, while she continues giving instructions to the group.

Along with adopting a plan that has not been created by her, the teacher carries out a class underlined by her personal style. Using the didactic proposal of the Strategy, she uses a margin of professional freedom to implement it. In order to solve tensions between the proposed plan and her appreciation about the students' competence, the teacher attributes Pepito the responsibility of the plan. The Pepito's messages constitute a means of controlling the program's execution but, at the same time, it is possible to qualify some activities like too easy: "¡but if everybody already knows it, Pepito!", or very complex: "how difficult is this!". Although she practically carries out all the proposed activities, introduces variations: she enlarges the numeric range of 10 to 25 for ordering and reading of the numbers, she only manipulates the objects in some of the actions that generate additive situations, and she separates the same collection of objects that previously had joined, proposing this way an intuitive anticipation of the inverse character of the addition and subtraction operations.

The teacher seems receptive from children propositions. She accepted their restlessness for the zero, allowing them to say the numeric sequence from this number, besides saying it from the 1. Also, she invites them to write additions on the blackboard, since some already know it, although the plan proposes they only write the result. She corrects them, introducing the equal sign, and she shows them how to write a

subtraction. She is interested to know where they learned what they know and she stimulates them to share their knowledge during the class, without moving away from her own plan.

The children keep up expectant. They are happy, relaxed, and willing to carry out the proposed tasks. Teacher encourage children to express themselves openly, as much responding to her questions as expressing their emotional reactions and communicating their appreciations concerning any topic. This opening is propitiated by the supposed spontaneity of Pepito's behavior. Indeed, through Pepito, the teacher verbalizes infantile necessities and emotions, during the class. Pepito carries out mischiefs, feels happy or sad, shows with vehemence its impulses and the teacher feigns to dialogue with him to become calm, instead of doing directly with her students. In order to avoid conflicts, the teacher attributes complicated decision making to Pepito, for example, what boy goes to the blackboard in a given moment.

The relationship between the teacher and the children contains as much guiding elements as permissiveness. The teacher, encouraging them with Pepito's tales, manages the class; this management is accepted by all the children. On the other hand, she is able to establish a grade of trust that allows children to express themselves confidently, saying what they know and what they don't know, or something that other boy told them, generating a climate of frankness and intellectual honesty.

Finally, there is a 40% of absent children. Although the teacher recognizes that this situation "make Pepito unhappy", she does not get discouraged and she works with the current children with the same enthusiasm that, without a doubt, she would deploy if she had complete attendance.

What kind of reform is expected, with practices as the one observed?

In the observed class it is operating the didactic proposal elaborated by the Strategy of Consultantship to the School. The class is part of a Didactic Unit where it is proposed a planning for several classes, at the end of which it is awaited that the students achieve certain learnings. Through these classes, the students face different types of mathematical tasks, they should elaborate procedures or techniques to undertake these tasks. The tasks, they are proposed by the teacher, but the techniques arise from what children know or can discern, in the moment to undertake them. Once the task was carried out, the teacher manages a discussion in which children expose their techniques and they compare them, according to the effectiveness regarding the proposed task, in order to choose one of them for the use of the whole community.

The tasks and their execution techniques constitute the practical component of the mathematical activity, but the school study would not be complete if it have not been included its theoretical component. Once children have a technique to carry out certain task, this late is modified changing their conditions to carry out, so that children will have to change the techniques which they undertake. The simple task of "add 1", can be replaced with "add to 1", requesting children determine the quantity of notebooks when the teacher puts 1 red notebook and then 6 blues. The technique of saying the

following number of the first term is no longer useful and it is very probable that the children need to count all the notebooks to give the answer. The commutativity resource will arise but, how to justify this property?

Along the Didactic Unit, starting from the sequence of proposed tasks and from the evolution of the techniques used to solve them, the teacher opens the discussion about the progress in mathematical knowledge of the group. The discussion about techniques, their reach (where they work) and their justification (why they work), it corresponds to a more theoretical level of the mathematical work that is carried out in the classrooms. The teacher stimulates the search of relationships among the used techniques and he guides the formulation of properties, concepts and theorems. Walking towards the theory, it supports the search of relationships among the acquired knowledge.

The reform of the teaching and learning practices that is expected, begins with the study and application of these Didactic Units by the teachers of a school, under the management of an experienced teacher and previously qualified in the frame of the Strategy, the "consulting teacher". The study begins with a process of problematization of teachers' knowledge in regard to the topic boarded in the Unit. The teachers face a problem or mathematical task and they solve it with the techniques they manage, later identifying the mathematical knowledge they put at stake or those they have reformulated, or acquired, during the process. Then they read the Didactic Unit that contains, besides specific proposals of activities to organize the work with the children and evaluate the achieved learnings, an extensive chapter to argument, from a didactic and mathematical perspective, the curricular decisions made by the authors of the Unit.

The following step for the appropriation of the didactic proposal, consists of applying the Unit studied in the course where each teacher teach. In this process, teachers are accompanied by the consulting teacher who attends some of their classes as participant observer, subordinating his participation to the support needs expressed by the teacher that manages the class. Afterwards, the consulting teacher gives feedback to the teacher who applied the Unit, in individual and in collective sessions, with the participation of all the teachers of the school. In these sessions it is very useful to have videorecords of the classes, in order to have a repeated observation and an analysis more objective.

Besides training teachers starting from the study and application of four Didactic Units in a school year, the consulting teacher interacts with the directive personnel of the school, in order to generate institutional conditions that support the study and appropriation of the didactic proposal by the teachers.

The Didactic Units contain propositions to organize the educational work during a brief period, one or two weeks. They foreshadow a learning process, providing a basic structure that needs to be complemented by each teacher. It is in this complementation where the teacher's master is evidenced since his class has been selected as example of a good practice. Besides modifying some of the proposed activities, she puts at stake her knowledge about how to delight and to make their students work. When operating

through consulting teachers, the Strategy of Consultantship to the School aspires to enrich its proposals with the exchange of experiences among the group of teachers that are beginning the study of the Didactic Units and an experienced teacher that has already studied them and perform some of them with their students, as in the case of the consulting teacher whose class we have analysed.

The Strategy of Consultantship to the School proposes fundamental changes in the paradigm that operates in our national educational system at the moment. It intends to change:

- A teaching focalized in the learning of concepts and mathematical procedures, to be changed by a teaching based on the study of problems.
- A teaching of isolated concepts, to be changed by the undertaking of articulated problems fields.
- The presentation of definitions and explanations from the teacher or from a text, to be changed by the collective construction of senses and mathematical meanings, assumed as a cooperative task.
- An activity few established, to be changed by other, based on arguments and justifications arised from the own children work, and that respects the consistency and mathematical rigourness.

References

- Bosch, M., L. Espinoza and Gascon J. (2003). El professor como director de procesos de estudio: análisis de organizaciones didácticas espontáneas. *Recherches in Didactique gives Mathématiques*, 23(1), 79-136.
- Brousseau, Guy (1990). Qué pueden aportar a los enseñantes los diferentes enfoques de la Didáctica de las Matemáticas? Primera Parte. *Enseñanzade las Ciencias*, 8(3), 259-267.
- Chevallard, Y (1999). L'analyse des pratique enseignantes en théorie anthropologique du didactique. *Recherches en Didactique del Mathématiques*. 19(2), 221-266.

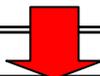
APPENDIX

1. Outline of the Didactic Unit
2. Plan for the first class
3. Work sheet 1
4. Work sheet 2
5. Work sheet 3
6. Work sheet 4.

SECOND DIDACTIC UNIT: *Additive Problems of Composition*

AWAITED LEARNINGS FROM PROGRAM

- *They associate the addition and subtraction operations with the actions of joining or separating sets and adding or removing objects, in situations that allow determining unknown information from available information. (Awaited learning 5, first semester).*
- *They manage mental calculation of additions and simple subtractions in the range 0 to 30. (Awaited learning 6, first semester).*



AWAITED LEARNINGS FOR THE UNIT

- They associate the addition with joining objects from two collections in only one.
- They associate the subtraction with separating objects from a collection in two collections.
- They solve additive problems associated to the actions of joining objects from two collections in only one or separating the objects from one collection in two.
- In some cases of additions and simple subtractions, they add or subtract for evoking results, that is, through mental calculation.

Previous learnings:

- Say the numeric sequence in upward and descending form, at least up to 10.
- Count objects, at least up to 10.
- Identify each one of the numbers up to 10 and copy them, for example, from a numbered ribbon.
- Locate a well-known number from the numbered ribbon, and continue saying the numeric sequence to locate the writing of another number.

Central ideas for this unit:

- Counting is a procedure that allows determining the quantity of objects that result from joining objects of two collections or separating objects of one collection in two parts.
- The action of joining objects from two collections in only one, it is associated to addition.
- The action of separating objects from one collection in two collections, it is associated to subtraction.
- Addition allows anticipating the quantity of objects that will result from joining objects of two collections.
- Subtraction allows anticipating the result of separating objects from one collection in two parts.
- In some specific cases, it is possible to anticipate the result of the actions through mental calculation

Traverse objective:

It will tend to develop in boys and girls, the self-confidence in the own possibilities of solving problems that imply results about certain actions.

Plan for the first class (90') Solving composition problems, in the range 0 to 10

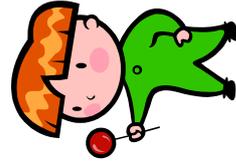
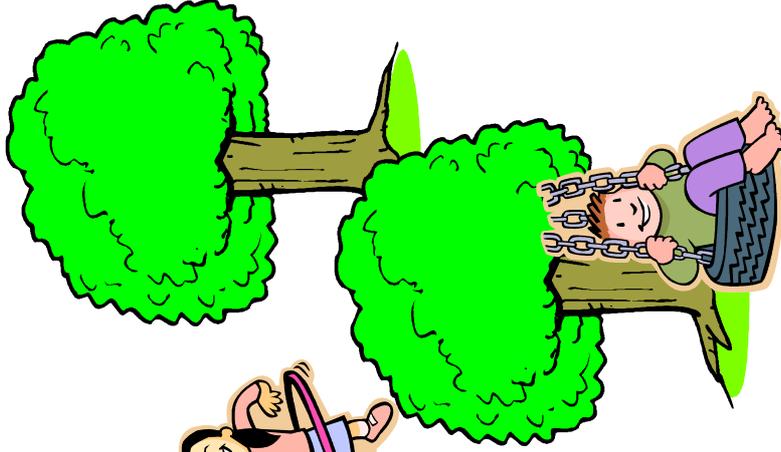
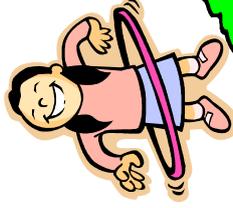
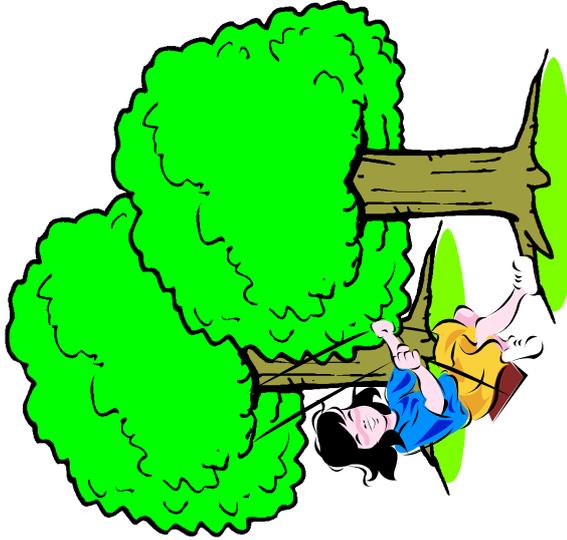
Materials: Objects in the classroom: school tools, colors cards, sticks, bottle covers, pupils, etc.

Mathematics Task	Activities	Evaluation
<p>They determine the quantity of objects that results from joining or separating actions, that the teacher verbally proposes, manipulating objects that are close to them.</p> <p>Numeric range up to 10.</p>	<p>Starting the class. Collective situation: The teacher exposes problems to children that allow him to be sure that all of them know:</p> <ul style="list-style-type: none"> - to say the numeric sequence in upward and descending form, at least up to 10 - to count at least up to 10 objects - to read the numbers up to 10 and to copy them from the numbered ribbon <p>For example:</p> <ul style="list-style-type: none"> - he asks a boy to begin the numeric sequence and others to continue this sequence up to 10; then he request other children to say the numeric sequence in descending form up to 1 or 0 - questions: how can we know how many children are there in this row?, how many are?, how many books are there on my desk?, etc. <p>Development of the class. Collective situation: For each problem that the teacher exposes, children manipulate the objects according to the proposed problem: they join or separate objects, according to the case. They always should give a complete oral answer and write the corresponding number. In relation to the relationship among the numbers, they are of two types:</p> <ol style="list-style-type: none"> 1. Some problems in which one of the collections has an object: Example 1: Ana has 7 sheets and Luis has one. They join their sheets. How many do they have? Example 2: Ana and Luis join 8 sheets. Ana has 1. How many does Luis have? 2. Others problems, they could include any couple of numbers (whose sum does not overcome 10): Example 3: There are 6 small books and 3 big books on the teacher's desk. How many books are there on the desk? Example 4: There are 9 books on the teacher's desk, 6 are small and the other ones are big. How many big books are there? <p>The teacher asks in each case: what are we doing with the objects?, what operation are we doing with the numbers?, how are we going to find the result?</p> <p>Work Sheet Task: Students work in work sheets 1, 2, 3 and 4. The teacher can add activities to those proposed in the work sheets.</p>	<p>The children performance according to appropriate tasks can suggest the teacher to propose activities like: say the sequence in descending form, count objects distributed in different forms, locate and copy numbers from the numbered ribbon.</p> <p>It is important that everybody join and separate objects correctly. There will be children that don't need to count all the objects; it is not necessary to be demanding with them. The different levels of students' performance will suggest the teacher to propose problems in a bigger range of numbers; joining or separating problems, etc. It is important that all children give a complete oral answer.</p> <p>Some students will need concrete material.</p>
	<p>Closing the class. At the end of this class, the teacher should systematize what they learned: to solve problems, a procedure is adding (if they have joined objects) or subtracting (if they have separated objects) through counting. In the cases in that a collection has only one object, the result is the following number or the previous number, according to the case.</p>	

Work Sheet 1

Name: _____

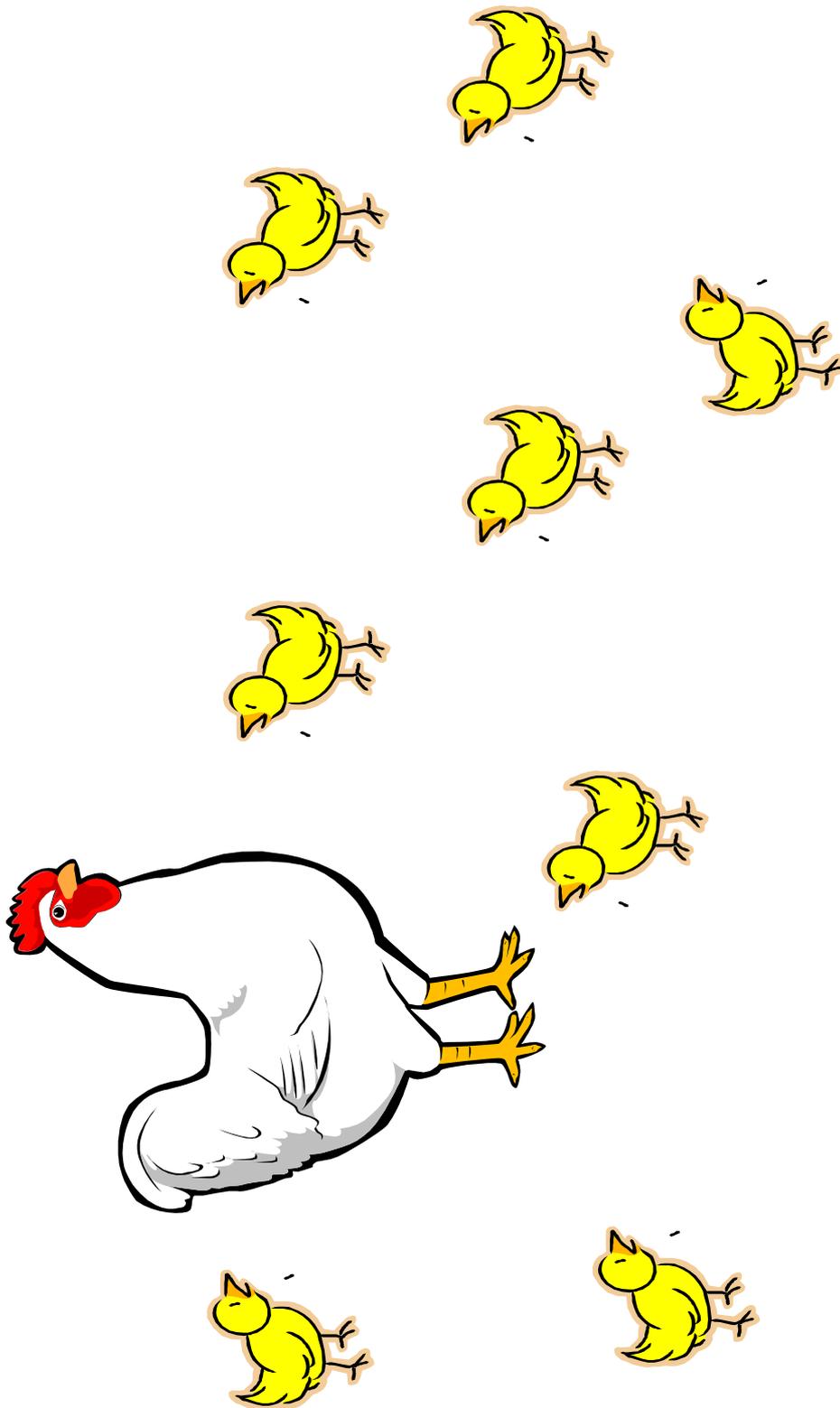
Answer verbally: How many girls are there in the square? How many boys are there?
Write into the box the total amount of boys and girls that there are in the square.
You can copy the number from the numbered ribbon.



Work Sheet 2

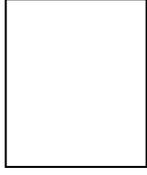
Name: _____

Respond verbally: How many birds are there on the sheet?
Write into the box how many chickens there are.
You can copy the number from the numbered ribbon.



Work Sheet 3

Name: _____



Inside the frame there are rectangular and triangular objects.

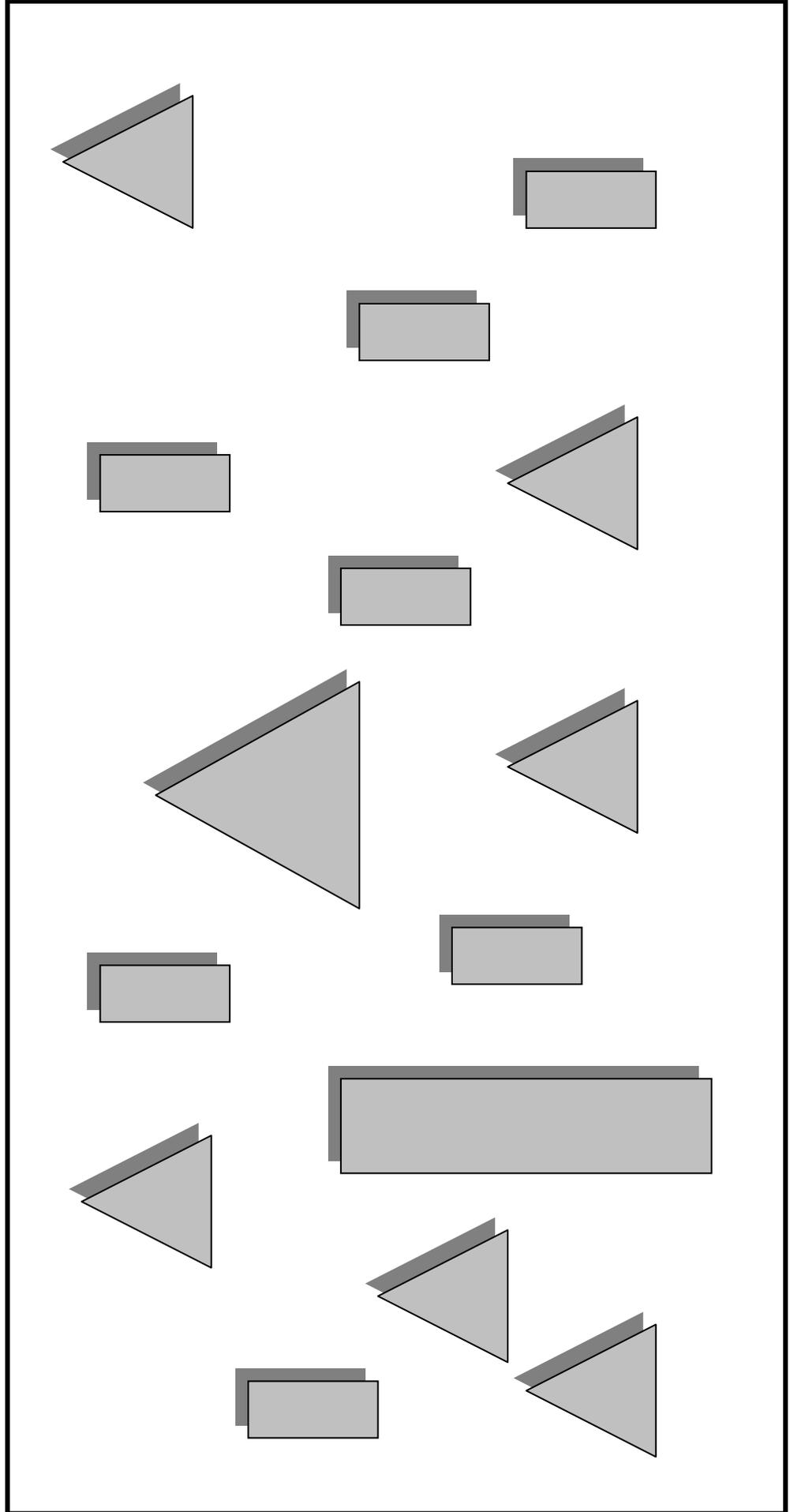
Answer verbally: How many rectangular small objects are there into the frame?

How many small triangular objects are there?

Write into the first box how many rectangular objects there are

Write into the second box how many triangular objects there are

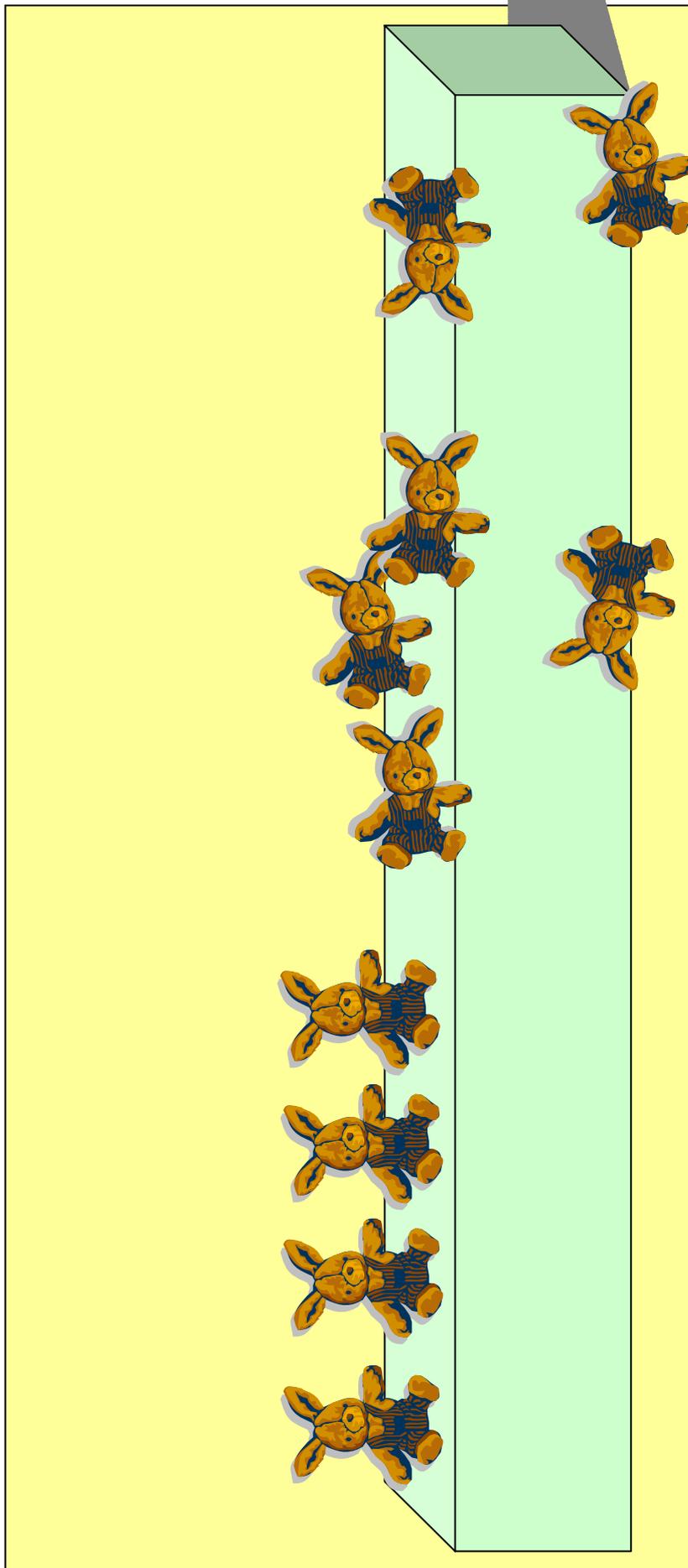
You can copy the numbers from the numbered ribbon.



Work Sheet 4

Name: _____

In a game 10 bears were placed and Ana made the first play.
Answer verbally: How many bears are standing?
Write into the box how many bears Ana throw away in her play.
You can copy the number from the numbered ribbon.



1st. Grade Second Unit